Solution of ISC Examination Questions 2020

SECTION-A

Solution 1

(i) Here, $*$ is defined by $a * b = |a - b|$

Let $a, b \in \mathbb{R}$ be any two elements.

Then, $a * b = |a - b|$

and $b * a = |b - a|$

$= |-(a - b)|$

$= |a - b|$

$\therefore \quad a * b = b * a$

Hence $*$ is commutative on $\mathbb{R}$.

Since, $a * b = |a - b|$

$\therefore \quad (-3) * 2 = |(-3) - 2|$

$= |-5| = 5.$

(ii) Let $\sec^{-1} 2 = \alpha \quad \Rightarrow \quad \sec\alpha = 2$

and $\text{cosec}^{-1} 3 = \beta \quad \Rightarrow \quad \text{cosec}\,\beta = 3$

Now, $\tan^2 (\sec^{-1} 2) + \cot^{-2} (\text{cosec}^{-1} 3)$

$= \tan^2\alpha + \cot^2\beta$

$= (\sec^2\alpha - 1) + (\text{cosec}^2\beta - 1)$

$= \sec^2\alpha + \text{cosec}^2\beta - 2$

$= (2)^2 + (3)^2 - 2 = 11.$

(iii) Here,
$$\Delta = \begin{vmatrix} 20 & a & b+c \\ 20 & b & c+a \\ 20 & c & a+b \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2$, we get
$$\Delta = \begin{vmatrix} 20 & a & a+b+c \\ 20 & b & a+b+c \\ 20 & c & a+b+c \end{vmatrix}$$

On taking 20 common from C_1 and $(a + b + c)$ common from C_3, we get
$$\Delta = 20(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$= 20(a + b + c) \cdot (0) = 0.$ [$\because$ C_1 and C_3 are identical.]

(iv) Here,
$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (2)(1)+(3)(-2) & (2)(-3)+(3)(4) \\ (5)(1)+(7)(-2) & (5)(-3)+(7)(4) \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

Corresponding elements of the above two matrices must be equal.

$$\therefore \quad x = 13.$$

(v) Here, $x^3 + y^3 = 3axy$

Differentiating both sides with respect to x, we get

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 3a\frac{d}{dx}(xy)$$

$$\Rightarrow \quad 3x^2 + 3y^2\frac{dy}{dx} = 3a\left\{x\frac{dy}{dx} + y\right\}$$

$$\Rightarrow \quad x^2 + y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$$

$$\Rightarrow \quad \frac{dy}{dx}(y^2 - ax) = (ay - x^2)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}.$$

(vi) Let 'a' denote the side of a cube and V the volume of cube at instant t.

Then, $V = a^3$

Differentiating both sides with respect to t, we get

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}$$

But, $\frac{da}{dt}$ = 10 cm per second

$$\therefore \quad \frac{dV}{dt} = 3a^2(10) = 30a^2$$

Now, $\left(\frac{dV}{dt}\right)_{a=5} = 30(5)^2 = 750$

Hence, the volume is increasing at the rate of 750 cu cm per second.

(vii) Here, $|x - 5| = \begin{cases} x-5 & \text{if } x-5 \geq 0 \\ -(x-5) & \text{if } x-5 < 0 \end{cases} = \begin{cases} x-5 & \text{if } x \geq 5 \\ -(x-5) & \text{if } x < 5 \end{cases}$

$\therefore$ The integral $\int_4^5 |x-5|\,dx$

$$= -\int_4^5 (x-5)\,dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_4^5$$

$$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{16}{2} - 20\right)\right]$$

$$= -\left[-\frac{25}{2} - (-12)\right]$$

$$= -\left(-\frac{1}{2}\right) = \frac{1}{2}.$$

(viii) Here, $y^2 = 4ax$ ...(1)

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = 4a$$

Substituting this value of $4a$ in (1) we get

$$y^2 = \left(2y\frac{dy}{dx}\right)x$$

$\Rightarrow$ $$y = 2x\frac{dy}{dx}$$

which is the required differential equation of the given curve.

(ix) Let E be the event defined as follows:

E : Drawing a white ball.

$\therefore$ $P(E) = \dfrac{5}{16}$ [$\because$ There are 16 balls out of which 5 are white.]

and $P(\overline{E}) = 1 - P(E)$

$$= 1 - \frac{5}{16} = \frac{11}{16}$$

$\therefore$ Required probability $= P(\text{no ball is white})$

$$= P(\overline{E})\,P(\overline{E})\,P(\overline{E})\,P(\overline{E})$$

$$= \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16} = \left(\frac{11}{16}\right)^4.$$

(x) Here, $P(A) = \dfrac{1}{2}$, $P(B) = p$ and $P(A \cup B) = \dfrac{3}{5}$.

As A and B are independent events

$\therefore$ $P(A \cap B) = P(A)\,P(B)$

$$= \frac{1}{2}(p) = \frac{p}{2}$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $$\frac{3}{5} = \frac{1}{2} + p - \frac{p}{2}$$

$\Rightarrow$ $$\frac{3}{5} = \frac{1}{2} + \frac{p}{2}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{3}{5} - \frac{1}{2}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{1}{10} \quad \Rightarrow \quad p = \frac{1}{5}.$$

Solution 2

Here, $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \dfrac{3x+4}{5x-7}, x \neq \dfrac{7}{5}$

and $g : \mathbb{R} \to \mathbb{R}$, defined by $g(x) = \dfrac{7x+4}{5x-3}, x \neq \dfrac{3}{5}$

$\therefore \quad g \circ f : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{3x+4}{5x-7}\right) \\ &= \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3} \\ &= \frac{7(3x+4)+4(5x-7)}{5(3x+4)-3(5x-7)} \\ &= \frac{41x}{41} = x.\end{aligned}$$

and $f \circ g : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{7x+4}{5x-3}\right) \\ &= \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7} \\ &= \frac{3(7x+4)+4(5x-3)}{5(7x+4)-7(5x-3)} \\ &= \frac{41x}{41} = x.\end{aligned}$$

Hence, $(g \circ f)(x) = (f \circ g)(x)$.

Solution 3

(a) Here, $$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$$

$$\Rightarrow \quad \cos^{-1}\left[\frac{x}{2}\cdot\frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2}\sqrt{1-\left(\frac{y}{3}\right)^2}\right] = \theta$$

$$\Rightarrow \quad \cos^{-1}\left[\frac{xy}{6} - \sqrt{\left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)}\right] = \theta$$

$$\Rightarrow \quad \frac{xy}{6} - \sqrt{1-\frac{x^2}{4} - \frac{y^2}{9} + \frac{x^2y^2}{36}} = \cos\theta$$

$$\Rightarrow \quad \frac{xy}{6} - \cos\theta = \sqrt{1-\frac{x^2}{4} - \frac{y^2}{9} + \frac{x^2y^2}{36}}$$

Squaring both sides, we get

$$\frac{x^2y^2}{36} + \cos^2\theta - 2\frac{xy}{6}\cdot\cos\theta = 1 - \frac{x^2}{4} - \frac{y^2}{9} + \frac{x^2y^2}{36}$$

$$\Rightarrow \quad -\frac{xy}{3}\cos\theta + \frac{x^2}{4} + \frac{y^2}{9} = 1 - \cos^2\theta$$

$$\Rightarrow \quad -12xy\cos\theta + 9x^2 + 4y^2 = 36\sin^2\theta$$

$$\Rightarrow \quad 9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta.$$

OR

(b) Here, $\cos(2\cos^{-1}x + \sin^{-1}x)$

$$= \cos(\cos^{-1}x + \cos^{-1}x + \sin^{-1}x)$$

$$= \cos\left(\cos^{-1}x + \frac{\pi}{2}\right)$$

$$= \cos\left(\frac{\pi}{2} + \cos^{-1}x\right)$$

$$= -\sin(\cos^{-1}x)$$

$$= -\sin\left[\sin^{-1}\sqrt{1-x^2}\right]$$

$$= -\sqrt{1-x^2}$$

$$= -\sqrt{1-\left(\frac{1}{5}\right)^2} \qquad \left(\text{at } x = \frac{1}{5}\right)$$

$$= -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}.$$

Solution 4

Here, $\quad$ L.H.S. $= \begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$

$$= \begin{vmatrix} x-p & p & q \\ p-x & x & q \\ 0 & q & x \end{vmatrix} \qquad [\text{On applying } C_1 \to C_1 - C_2]$$

$$= (x-p)\begin{vmatrix} 1 & p & q \\ -1 & x & q \\ 0 & q & x \end{vmatrix}$$ [On taking $(x - p)$ common from C_1]

$$= (x-p)\begin{vmatrix} 1 & p & q \\ 0 & x+p & 2q \\ 0 & q & x \end{vmatrix}$$ [On applying $R_2 \to R_2 + R_1$]

$$= (x-p)[(x+p)x - 2q^2]$$ [Expanding along C_1]

$$= (x-p)(x^2 + px - 2q^2) = \text{R.H.S.}$$

Solution 5

Here, $f(x) = -1 + \cos x$ on $[0, 2\pi]$

Clearly $f(x)$ is defined for all $x \in [0, 2\pi]$

Since, constant function and cosine function, both are continuous and differentiable everywhere

Also, the difference of continuous functions is continuous and the difference of differentiable functions is differentiable.

Therefore, (i) $f(x)$ is continuous on $[0, 2\pi]$.

(ii) $f(x)$ is differentiable on $(0, 2\pi)$.

$\therefore \quad f'(x) = -\sin x$

(iii) $f(0) = 0$, $f(2\pi) = 0$

So, $f(0) = f(2\pi)$

Since, the conditions of Rolle's theorem are satisfied, then there must exist at least one value of $c \in (0, 2\pi)$ such that

$$f'(c) = 0$$

$\Rightarrow \quad -\sin c = 0$

$\Rightarrow \quad \sin c = 0$

$\Rightarrow \quad c = 0, \pi, 2\pi$

$\Rightarrow \quad c = \pi \in (0, 2\pi)$

Hence, Rolle's theorem is verified for $f(x) = -1 + \cos x$ on $[0, 2\pi]$.

Solution 6

Here, $$y = e^{m \sin^{-1} x} \quad \text{...(1)}$$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = e^{m \sin^{-1} x} \frac{d}{dx}(m \sin^{-1} x)$$

$\Rightarrow$ $$\frac{dy}{dx} = y\left(\frac{m}{\sqrt{1-x^2}}\right)$$ [Using (1)]

$\Rightarrow$ $$\sqrt{1-x^2}\,\frac{dy}{dx} = my \quad \text{...(2)}$$

Differentiating both sides with respect to x, we get

$$\sqrt{1-x^2}\ \frac{d^2y}{dx^2}+\frac{dy}{dx}\left(-\frac{x}{\sqrt{1-x^2}}\right)=m\,\frac{dy}{dx}$$

$$\Rightarrow \qquad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m\sqrt{1-x^2}\ \frac{dy}{dx}$$

$$\Rightarrow \qquad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y. \qquad \text{[Using (2)]}$$

Solution 7

(a) Here, the given curve is $y^2 = px^3 + q$ ...(1)

Also, the given point on the curve is (2, 3)

$$\therefore \qquad (3)^2 = p(2)^3 + q$$

$$\Rightarrow \qquad 9 = 8p + q \qquad ...(2)$$

Differentiating both sides of (1), with respect to x, we get

$$2y\frac{dy}{dx} = p(3x^2)$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{3p}{2}\left(\frac{x^2}{y}\right)$$

$\therefore$ The slope of tangent at (2, 3) is

$$\left(\frac{dy}{dx}\right)_{(2,\,3)} = \frac{3p}{2}\left(\frac{(2)^2}{3}\right) = 2p$$

Also, the slope of given tangent ($y = 4x + 7$) is 4.

$$\therefore \qquad 2p = 4 \quad \Rightarrow \quad p = 2$$

Substituting $p = 2$ in (2), we get

$$9 = 8(2) + q \quad \Rightarrow \quad q = -7$$

Hence, $p = 2$ and $q = -7$.

OR

(b) Here, $\lim\limits_{x\to 0}\left[\dfrac{xe^x-\log(1+x)}{x^2}\right]$ $\qquad \left[\dfrac{0}{0}\text{ form}\right]$

$$= \lim_{x\to 0}\left[\frac{\{xe^x-\log(1+x)\}'}{(x^2)'}\right] \qquad \text{[By L' Hôpital's Rule]}$$

$$= \lim_{x\to 0}\left[\frac{(xe^x+e^x)-\dfrac{1}{1+x}}{2x}\right] \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to 0}\left[\frac{\left\{(xe^x+e^x)-\dfrac{1}{1+x}\right\}'}{(2x)'}\right] \qquad \text{[By L' Hôpital's Rule]}$$

$$= \lim_{x\to 0}\left[\frac{xe^x + e^x\cdot 1 + e^x + \dfrac{1}{(1+x)^2}}{2}\right]$$

$$= \frac{0+1+1+1}{2} = \frac{3}{2}.$$

Solution 8

(a) Here, $\displaystyle\int \frac{1}{\sqrt{5x-4x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{\frac{5}{4}x - x^2}}\,dx$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\frac{5}{4}x - x^2 + \frac{25}{64} - \frac{25}{64}}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\frac{25}{64} - \left(x^2 - \frac{5x}{4} + \frac{25}{64}\right)}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{5}{8}\right)^2 - \left(x - \frac{5}{8}\right)^2}}\,dx$$

$$= \frac{1}{2}\sin^{-1}\left[\frac{\left(x - \frac{5}{8}\right)}{\frac{5}{8}}\right] + C$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-5}{5}\right) + C.$$

OR

(b) Let $\displaystyle I = \int \sin^3 x \cos^4 x\, dx = \int \sin x \sin^2 x \cos^4 x\, dx$

$$= \int \sin x\,(1-\cos^2 x)\cos^4 x\, dx$$

Put $\cos x = y \Rightarrow \sin x\, dx = -dy$

$\therefore$
$$I = -\int (1-y^2)y^4\, dy$$

$$= \int (y^6 - y^4)\, dy$$

$$= \frac{y^7}{7} - \frac{y^5}{5} + C$$

$$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C.$$

Solution 9

Here, $(1+x^2)\dfrac{dy}{dx} = 4x^2 - 2xy$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

which is linear differential equation in y of the form $\dfrac{dy}{dx} + Py = Q$ with $P = \dfrac{2x}{1+x^2}$ and $Q = \dfrac{4x^2}{1+x^2}$.

Now, integrating factor, $I.F. = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx}$

$$= e^{\log(1+x^2)}$$

$$= 1 + x^2$$

$\therefore$ General solution of given differential equation is

$$y(I.F.) = \int Q\,(I.F.)\,dx + C$$

$$\Rightarrow \quad y(1+x^2) = \int \frac{4x^2}{1+x^2}(1+x^2)\,dx + C$$

$$\Rightarrow \quad y(1+x^2) = \int 4x^2 dx + C$$

$$\Rightarrow \quad y(1+x^2) = 4\left(\frac{x^3}{3}\right) + C$$

which is the required solution.

Solution 10

Let E, F and G be the events defined as follows:

E : A hits the target.

F : B hits the target.

G : C hits the target.

$$\therefore \quad P(E) = \frac{5}{6}, \qquad P(\overline{E}) = 1 - P(E) = 1 - \frac{5}{6} = \frac{1}{6}$$

$$P(F) = \frac{4}{5}, \qquad P(\overline{F}) = 1 - P(F) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$P(G) = \frac{3}{4}, \qquad P(\overline{G}) = 1 - P(G) = 1 - \frac{3}{4} = \frac{1}{4}$$

Events E, F and G are independent.

(i) Required probability = $P(A$ hits, B hits, C fails) + $P(A$ hits, B fails, C hits,) + $P(A$ fails, B hits, C hits)

$$= P(E)P(F)P(\overline{G}) + P(E)P(\overline{F})P(G) + P(\overline{E})P(F)P(G)$$

$$= \left(\frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}\right) + \left(\frac{5}{6}\times\frac{1}{5}\times\frac{3}{4}\right) + \left(\frac{1}{6}\times\frac{4}{5}\times\frac{3}{4}\right)$$

$$= \frac{20}{120} + \frac{15}{120} + \frac{12}{120} = \frac{47}{120}.$$

(ii) Required probability = P(at least one person hits the target)

$= 1 - P(\text{none of } A, B, C \text{ hits the target})$

$= 1 - P(\overline{E})\, P(\overline{F})\, P(\overline{G})$ [$\because$ Events E, F and G are independent so, $\overline{E}$, $\overline{F}$ and $\overline{G}$ are also independent.]

$= 1 - \left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)$

$= 1 - \frac{1}{120} = \frac{119}{120}$.

Solution 11

Here, the given system of linear equations is

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

This system of equations can be written as

$$AX = B$$

where, $$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Now, $$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 1(-1-2) + 2(2-0) + 0(-4-0) = 1 \neq 0$$

As $|A| \neq 0$, A^{-1} exists and the given system of equations has a unique solution

$$X = A^{-1} B$$

Let A_{ij} denote cofactor of a_{ij} in $A = [a_{ij}]$.

So, the cofactors of the elements of A are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} = -3, \quad A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} = 2, \quad A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 0 \\ -1 & -1 \end{vmatrix} = 2,$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -2, \quad A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, \quad A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = 1,$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} = -4, \quad A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = 2, \quad A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = 3.$$

$\therefore$ $$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

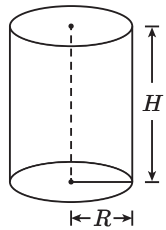
Now, $$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$\therefore \qquad X = A^{-1} B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \qquad x = 0, y = -5, z = -3$$

Hence, the required solution is $x = 0$, $y = -5$, $z = -3$.

Solution 12



(a) Let R be the radius of base and H be the height of the cylinder.

$\therefore$ Surface area of cylinder, $S = 2\pi RH + 2\pi R^2$

$$\Rightarrow \qquad H = \frac{S - 2\pi R^2}{2\pi R} \qquad ...(1)$$

and its volume, $V = \pi R^2 H$

$$= \pi R^2 \left(\frac{S - 2\pi R^2}{2\pi R} \right) \qquad \text{[Using (1)]}$$

$$= \frac{1}{2}(SR - 2\pi R^3)$$

$$\therefore \qquad \frac{dV}{dR} = \frac{1}{2}(S - 6\pi r^2)$$

Now, $\dfrac{dV}{dR} = 0 \Rightarrow \dfrac{1}{2}(S - 6\pi R^2) = 0$

$$\Rightarrow \qquad S = 6\pi R^2$$

$$\Rightarrow \qquad R = \sqrt{\frac{S}{6\pi}}$$

Also, $\dfrac{d^2V}{dR^2} = -6\pi R$

$$\Rightarrow \qquad \left(\frac{d^2V}{dR^2}\right)_{R=\sqrt{S/6\pi}} = -6\pi\left(\sqrt{\frac{S}{6\pi}}\right) < 0$$

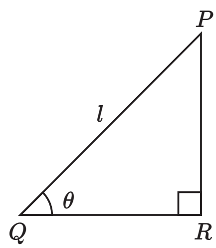
$\therefore$ Volume is maximum when $R = \sqrt{\dfrac{S}{6\pi}}$ or $S = 6\pi R^2$

$\therefore$ Putting $S = 6\pi R^2$ in (1), we get

$$H = \frac{6\pi R^2 - 2\pi R^2}{2\pi R} = 2R \quad \text{or} \quad R = \frac{H}{2}$$

Hence, the volume of cylinder is maximum when its radius is equal to half of its height.

OR



(b) Let l be the length of hypotenuse and $\angle PQR = \theta$ $\left(0 < \theta < \dfrac{\pi}{2}\right)$ of a right-angled triangle PQR.

$$\therefore \qquad QR = l\cos\theta \quad \text{and} \quad PR = l\sin\theta$$

$\therefore$ Area of ΔABC,

$$A = \frac{1}{2}(l\cos\theta)(l\sin\theta)$$

$$= \frac{1}{2}(l^2\sin\theta\cos\theta)$$

$$= \frac{l^2}{4}(\sin 2\theta)$$

Also, $$\frac{dA}{d\theta} = \frac{l^2}{4}(2\cos 2\theta) = \frac{l^2}{2}\cos 2\theta$$

and $$\frac{d^2A}{d\theta^2} = \frac{l^2}{2}(-2\sin 2\theta) = -l^2\sin 2\theta$$

Now, $$\frac{dA}{d\theta} = 0 \Rightarrow \frac{l^2}{2}\cos 2\theta = 0$$

$$\Rightarrow \cos 2\theta = 0$$

$$\Rightarrow 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Also, $$\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\frac{\pi}{4}} = -l^2\sin 2\left(\frac{\pi}{4}\right)$$

$$= -l^2\sin\frac{\pi}{2} = -l^2 < 0$$

$\therefore$ Area is maximum, when $\theta = \frac{\pi}{4}$.

$\therefore$ $QR = l\cos\frac{\pi}{4} = \frac{l}{\sqrt{2}}$ and $PR = l\sin\frac{\pi}{4} = \frac{l}{\sqrt{2}} \Rightarrow QR = PR$

Hence, the triangle is isosceles.

Solution 13

(a) Let $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Put $x = \cos\theta \Rightarrow dx = -\sin\theta\,d\theta$

$$\therefore\quad I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta)d\theta = \int \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}(-\sin\theta)d\theta$$

$$= -\int \tan^{-1}\left(\tan\frac{\theta}{2}\right)\sin\theta\,d\theta$$

$$= -\int \frac{\theta}{2}\sin\theta\,d\theta$$

$$= -\frac{1}{2}\int \theta\sin\theta\,d\theta$$

$$= -\frac{1}{2}\left[\theta\int \sin\theta\, d\theta - \int\left(\frac{d}{d\theta}(\theta)\int \sin\theta\, d\theta\right)d\theta\right]$$

$$= -\frac{1}{2}\left[\theta(-\cos\theta) - \int 1(-\cos\theta)\, d\theta\right]$$

$$= -\frac{1}{2}\left[-\theta\cos\theta + \int \cos\theta\, d\theta\right]$$

$$= -\frac{1}{2}[-\theta\cos\theta + \sin\theta] + C$$

$$= \frac{1}{2}[\theta\cos\theta - \sin\theta] + C$$

$$= \frac{1}{2}\left[(\cos^{-1}x)\, x - \sqrt{1-\cos^2\theta}\right] + C$$

$$= \frac{1}{2}\left[x\cos^{-1}x - \sqrt{1-x^2}\right] + C.$$

OR

(b) Let $I = \int \frac{2x+7}{x^2-x-2}\,dx = \int \frac{2x+7}{(x-2)(x+1)}\,dx$ ...(1)

Put $\frac{2x+7}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$ ...(2)

$\Rightarrow$ $2x + 7 = A(x + 1) + B(x - 2)$

$\Rightarrow$ $2x + 7 = (A + B)x + (A - 2B)$

Comparing coefficients of x and constant terms on both sides, we get

$$A + B = 2 \quad \text{and} \quad A - 2B = 7$$

Solving these equations, we get

$$A = \frac{11}{3}, \qquad B = -\frac{5}{3}$$

Putting values of A and B in (2), we get

$$\frac{2x+7}{(x-2)(x+1)} = \frac{11}{3(x-2)} - \frac{5}{3(x+1)} \quad ...(3)$$

From (1) and (3), we get

$$I = \frac{11}{3}\int \frac{1}{x-2}\,dx - \frac{5}{3}\int \frac{1}{x+1}\,dx$$

$$= \frac{11}{3}\log|x-2| - \frac{5}{3}\log|x+1| + C.$$

Solution 14

Let p denote the probability that a bulb will fuse after 150 days and q denote the probability that the bulb will not fuse after 150 days.

$\therefore$ $p = 0.05 = \frac{1}{20}$, $q = 1 - \frac{1}{20} = \frac{19}{20}$, $n = 5$

Probability function $P(X = x) = {}^5C_x\, p^x q^{5-x}$

(i) Probability that none of the bulbs will fuse after 150 days

i.e., $$P(X = 0) = {}^5C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{5-0} = \left(\frac{19}{20}\right)^5.$$

(ii) Probability that not more than one bulb will fuse after 150 days

i.e., $$P(X \le 1) = P(X = 0) + P(X = 1)$$
$$= {}^5C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{5-0} + {}^5C_1\left(\frac{1}{20}\right)^1\left(\frac{19}{20}\right)^{5-1}$$
$$= \left(\frac{19}{20}\right)^5 + 5\left(\frac{1}{20}\right)\left(\frac{19}{20}\right)^4$$
$$= \left[\frac{19}{20} + \frac{5}{20}\right]\left(\frac{19}{20}\right)^4 = \frac{24}{20}\left(\frac{19}{20}\right)^4 = \frac{6}{5}\left(\frac{19}{20}\right)^4.$$

(iii) Probability that more than one bulb will fuse after 150 days

i.e., $$P(X > 1) = 1 - P(X \le 1)$$
$$= 1 - \frac{6}{5}\left(\frac{19}{20}\right)^4.$$

(iv) Probability that at least one bulb will fuse after 150 days

i.e., $$P(X \ge 1) = 1 - P(X = 0)$$
$$= 1 - \left(\frac{19}{20}\right)^5.$$

SECTION-B

Solution 15

(a) Let $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$. Then, unit vector in the direction of $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{|\hat{i} - 2\hat{j} - 2\hat{k}|}$$
$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}} = \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$$

$\therefore$ Required vector $= 18\vec{a} = 18 \cdot \dfrac{(\hat{i} - 2\hat{j} - 2\hat{k})}{3} = 6(\hat{i} - 2\hat{j} - 2\hat{k})$.

(b) Here, the given lines are

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4} \text{ and } \frac{x-1}{5} = \frac{y+2}{2} = \frac{z-1}{-5}$$

The direction ratios of these lines are 2, 5, 4 and 5, 2, –5.

Let $a_1 = 2, b_1 = 5, c_1 = 4$ and $a_2 = 5, b_2 = 2, c_2 = -5$

If α is the angle between the two lines, then

$$\cos\alpha = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left| \frac{(2)(5)+(5)(2)+(4)(-5)}{\sqrt{(2)^2+(5)^2+(4)^2}\sqrt{(5)^2+(2)^2+(-5)^2}} \right|$$

$$= \left| \frac{10+10-20}{\sqrt{45}\sqrt{54}} \right| = 0$$

$\Rightarrow \qquad \alpha = 90°$

Hence, the required angle is 90°.

(c) Here, the direction ratios of the line joining the points (4, 5, 0) and (1, –2, 4) are

$$1-4,\ -2-5,\ 4-0, \text{ i.e., } -3, -7, 4$$

Equation of any plane which is perpendicular to a line whose direction ratios are –3, –7, 4 is

$$-3x - 7y + 4z = \lambda \qquad ...(1)$$

This plane passes through (2, –3, 1)

$\therefore \qquad -3(2) - 7(-3) + 4(1) = \lambda$

$\Rightarrow \qquad \lambda = 19$

Substituting this value of λ in (1), we get

$$-3x - 7y + 4z = 19$$

or $\qquad 3x + 7y - 4z + 19 = 0$

which is the required equation of plane.

Solution 16

(a) Here, $\vec{a} \cdot \left[(\vec{b}+\vec{c})\times(\vec{a}+3\vec{b}+4\vec{c})\right]$

$$= \vec{a} \cdot \left[(\vec{b}\times\vec{a})+3(\vec{b}\times\vec{b})+4(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})+3(\vec{c}\times\vec{b})+4(\vec{c}\times\vec{c})\right]$$

$$= \vec{a} \cdot \left[(\vec{b}\times\vec{a})+4(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})+3(\vec{c}\times\vec{b})\right] \qquad \left[\because\ \vec{b}\times\vec{b} = \vec{0},\ \vec{c}\times\vec{c} = \vec{0}\right]$$

$$= \vec{a}\cdot(\vec{b}\times\vec{a})+4\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{c}\times\vec{a})+3\vec{a}\cdot(\vec{c}\times\vec{b})$$

$$= 0+4\vec{a}\cdot(\vec{b}\times\vec{c})+0-3\vec{a}\cdot(\vec{b}\times\vec{c}) \qquad \left[\because\ \vec{c}\times\vec{b} = -\vec{b}\times\vec{c}\right]$$

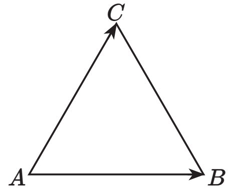
$$= 4\vec{a}\cdot(\vec{b}\times\vec{c})-3\vec{a}\cdot(\vec{b}\times\vec{c})$$

$$= \vec{a}\cdot(\vec{b}\times\vec{c})$$

$$= \left[\vec{a}\ \vec{b}\ \vec{c}\right].$$

OR

(b) Given a ΔABC with vertices $A(3, -1, 2)$ $B(1, -1, -3)$ and $C(4, -3, 1)$.



Then, $\qquad \overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= (\hat{i}-\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k}) = -2\hat{i}-5\hat{k}.$$

and $\qquad \overrightarrow{AC}$ = Position vector of C – Position vector of A

$$= (4\hat{i}-3\hat{j}+\hat{k})-(3\hat{i}-\hat{j}+2\hat{k}) = \hat{i}-2\hat{j}-\hat{k}.$$

Now, $\quad \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$

$$= \hat{i}(0-10) - \hat{j}(2+5) + \hat{k}(4-0) = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

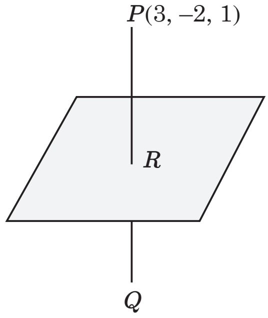
$$\Rightarrow \quad |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$\therefore$ Area of triangle $ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{165}$ sq units.

Solution 17

(a) Let R be the foot of the perpendicular from $P(3, -2, 1)$ on the plane

$$3x - y + 4z = 2 \qquad \text{...(1)}$$



$\therefore$ Direction ratios of the normal to the plane are $3, -1, 4$.

$\therefore$ Direction ratios of line perpendicular to plane (1) are $3, -1, 4$.

So, the equation of line passing through $P(3, -2, 1)$ and perpendicular to plane (1) is

$$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} \qquad \text{...(2)}$$

Let $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = \lambda$

Then, $x = 3\lambda + 3, \quad y = -\lambda - 2, \quad z = 4\lambda + 1$

$\therefore$ Any point on line (2) is $R(3\lambda + 3, -\lambda - 2, 4\lambda + 1)$

If R lies on the plane (1), then

$3(3\lambda + 3) - (-\lambda - 2) + 4(4\lambda + 1) = 2$

$\Rightarrow \quad 26\lambda + 13 = 0$

$\Rightarrow \quad \lambda = -\frac{1}{2}$

$\therefore$ The coordinates of the foot of perpendicular, i.e., the point R are

$$\left(3\cdot\left(-\frac{1}{2}\right)+3, -\left(-\frac{1}{2}\right)-2, 4\left(-\frac{1}{2}\right)+1\right), \text{ i.e., } \left(\frac{3}{2}, -\frac{3}{2}, -1\right).$$

Let $Q(\alpha, \beta, \gamma)$ be the image of point $P(3, -2, 1)$ in the plane (1), then R is the mid-point of PQ.

$$\therefore \quad \frac{3+\alpha}{2} = \frac{3}{2}, \quad \frac{-2+\beta}{2} = -\frac{3}{2}, \quad \frac{1+\gamma}{2} = -1$$

$$\Rightarrow \quad \alpha = 0, \quad \beta = -1, \quad \gamma = -3.$$

Hence, the image of point P in the given plane is $(0, -1, -3)$.

OR

(b) Here, the given lines are

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \text{ and } \frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$$

$\therefore$ Direction ratios of the given lines are $1, 2, 3$ and $-3, 2, 5$.

Let direction ratios of required line be a, b, c.

Since, the required line is perpendicular to both given lines.

$\therefore \quad 1\cdot a + 2\cdot b + 3\cdot c = 0 \qquad \text{...(1)}$

and $\quad -3\cdot a + 2\cdot b + 5\cdot c = 0 \qquad \text{...(2)}$

On solving (1) and (2), we get

$$\frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

i.e., $$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = \lambda\,(\text{say})$$

$\Rightarrow$ $$a = 4\lambda,\ b = -14\lambda,\ c = 8\lambda$$

$\therefore$ Direction ratios of the required line are $4\lambda, -14\lambda, 8\lambda$, i.e., 2, –7, 4.

Also, given that the required line passes through (–1, 3, –2).

Hence, equation of required line is

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}.$$

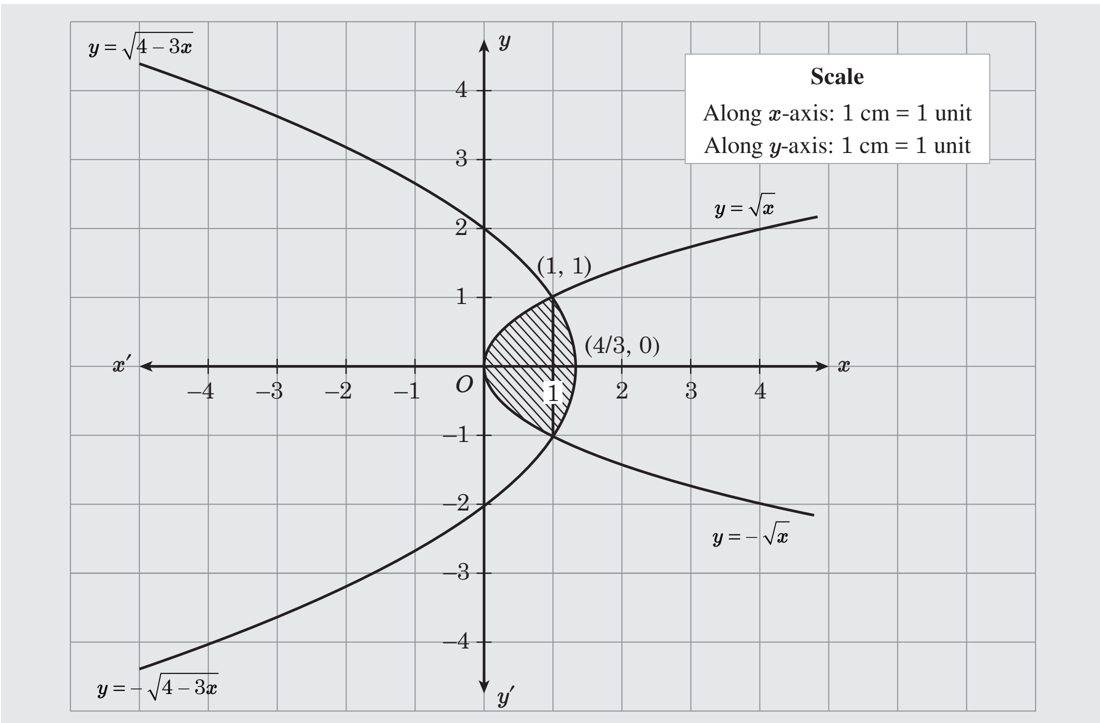
Solution 18

The curve $y^2 = x$ represents a parabola with vertex at origin and it opens on the right.

The curve $y^2 = 4 - 3x$

i.e., $y^2 = -3\left(x - \frac{4}{3}\right)$

represents a parabola with vertex $\left(\frac{4}{3}, 0\right)$ and it opens on the left.



Also, the two curves intersect at (1, 1) in the first quadrant

$\therefore$ Required area $= 2\left[\int_0^1 \left(\sqrt{x}-0\right)dx + \int_1^{4/3}\left(\sqrt{4-3x}-0\right)dx\right]$ (Required area is symmetrical about x-axis.)

$$= 2\left[\int_0^1 \sqrt{x}\,dx + \int_1^{4/3}\sqrt{4-3x}\,dx\right]$$

$$= 2\left[\frac{2}{3}\cdot x^{3/2}\right]_0^1 + 2\left[-\frac{2}{9}(4-3x)^{3/2}\right]_1^{4/3}$$

$$= \frac{4}{3}(1-0) - \frac{4}{9}(0-1)$$

$$= \frac{4}{3}+\frac{4}{9} = \frac{16}{9}$$

Hence, required area = $\frac{16}{9}$ sq units.

SECTION-C

Solution 19

(a) Here, cost function, $C(x) = 35x + 250$

Revenue function, $R(x) = 60x$

(i) Profit function, $P(x) = R(x) - C(x)$

$$= 60x - (35x + 250)$$

$$= 25x - 250$$

(ii) At break even point, $P(x) = 0$

$\Rightarrow \quad 25x - 250 = 0$

$\Rightarrow \quad x = 10$

Hence, the break even point is $x = 10$.

(b) Here, $R(x) = 100x - x^2 - x^3$

(i) Demand function $= \frac{R(x)}{x}$

$$= \frac{100x - x^2 - x^3}{x}$$

$$= 100 - x - x^2.$$

(ii) Marginal revenue function $= \frac{d}{dx}R(x)$

$$= \frac{d}{dx}(100x - x^2 - x^3)$$

$$= 100 - 2x - 3x^2.$$

(c) We know that the regression lines pass through $(\overline{x}, \overline{y})$.

$\therefore \quad 4\overline{x} - 2\overline{y} = 4$ and $2\overline{x} - 3\overline{y} + 6 = 0$

Solving these two equations simultaneously, we get $\overline{x} = 3$ (the mean of x) and $\overline{y} = 4$ (the mean of y).

Solution 20

(a) Here, $r = 0.6$, $\overline{x} = 10$, $\overline{y} = 20$, $\sigma_x^2 = 225$ and $\sigma_y^2 = 400$

Now, $\sigma_x^2 = 225 \Rightarrow \sigma_x = 15$ and $\sigma_y^2 = 400 \Rightarrow \sigma_y = 20.$

$$\therefore \quad b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.6\left(\frac{15}{20}\right) = \frac{9}{20} \quad \text{and} \quad b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.6\left(\frac{20}{15}\right) = \frac{4}{5}$$

(i) The regression equation of y on x is

$$y - 20 = \frac{4}{5}(x - 10)$$

$$\Rightarrow \quad 5y - 4x = 60$$

The regression equation of x on y is

$$x - 10 = \frac{9}{20}(y - 20)$$

$$\Rightarrow \quad 20x - 9y = 20$$

(ii) Putting $x = 2$ in regression equation of y on x, we get

$$5y - 4(2) = 60 \quad \Rightarrow \quad 5y = 68$$

$$\Rightarrow \quad y = \frac{68}{5} \text{ or } 13\frac{3}{5}.$$

OR

(b) We have the following table:

x_i	y_i	$x_i\, y_i$	x_i^2	y_i^2
2	8	16	4	64
6	8	48	36	64
4	5	20	16	25
7	6	42	49	36
5	2	10	25	4
$\Sigma x_i = 24$	$\Sigma y_i = 29$	$\Sigma x_i y_i = 136$	$\Sigma x_i^2 = 130$	$\Sigma y_i^2 = 193$

Here, $n = 5$.

$$\therefore \quad b_{yx} = \frac{n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{5(136) - (24)(29)}{5(130) - (24)^2}$$

$$= \frac{680 - 696}{650 - 576}$$

$$= -\frac{16}{74} = -\frac{8}{37}$$

$$b_{xy} = \frac{n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)}{n\Sigma y_i^2 - (\Sigma y_i)^2} = \frac{5(136) - (24)(29)}{5(193) - (29)^2}$$

$$= \frac{680 - 696}{965 - 841}$$

$$= -\frac{16}{124} = -\frac{4}{31}$$

We have, $r^2 = b_{yx} \cdot b_{xy} = \left(-\frac{8}{37}\right)\left(-\frac{4}{31}\right) = \frac{32}{1147}$

Hence, $r = -\sqrt{\frac{32}{1147}} = -0.17.$ $[\because r < 0, \text{ as } b_{yx}, b_{xy} < 0]$

Solution 21

(a) **(i)** Here, marginal cost function, MC = $30 + 2x$

$\therefore$ The cost function is given by

$$C(x) = \int \text{MC}\, dx = \int (30 + 2x)\, dx$$

$$= 30x + x^2 + \lambda \quad ...(1)$$

Given fixed cost is ₹ 200, when $x = 0$

$\therefore$ $200 = 3(0) + (0)^2 + \lambda \Rightarrow \lambda = 200$

Putting $\lambda = 200$ in (1), we get

$$C(x) = 30x + x^2 + 200$$

Hence, total cost function, $C(x) = 30x + x^2 + 200$.

(ii) The cost of increasing output from 100 to 200 units is

$$= C(200) - C(100)$$

$$= [30(200) + (200)^2 + 200] - [30(100) + (100)^2 + 200]$$

$$= 46200 - 13200 = 33000$$

Hence, the required cost is ₹ 33000.

OR

(b) Here, cost function, $C(x) = \frac{1}{3}x^3 - 5x^2 + 30x - 15$

and revenue function, $R(x) = 6x$

$\therefore$ Profit function, $P(x) = R(x) - C(x)$

$$= 6x - \left(\frac{1}{3}x^3 - 5x^2 + 30x - 15\right)$$

$$= -\frac{1}{3}x^3 + 5x^2 - 24x + 15$$

$\therefore$ $$\frac{dP}{dx} = -\frac{1}{3}\cdot(3x^2) + 10x - 24$$

$$= -x^2 + 10x - 24$$

and $$\frac{d^2P}{dx^2} = -2x + 10$$

Now, $$\frac{dP}{dx} = 0 \Rightarrow -x^2 + 10x - 24 = 0$$
$$\Rightarrow x^2 - 10x + 24 = 0$$
$$\Rightarrow (x-4)(x-6) = 0$$
$$\Rightarrow x = 4, 6$$

Also, $$\left(\frac{d^2P}{dx^2}\right)_{x=4} = -2(4) + 10 = 2 > 0$$

$$\left(\frac{d^2P}{dx^2}\right)_{x=6} = -2(6) + 10 = -2 < 0$$

Hence, profit is maximum when $x = 6$.

Solution 22

Let x be the number of half sleeves shirts and y be the number of full sleeves shirts to be manufactured per week.

Let total profit = ₹ Z

We can represent the given L.P.P. in the following tabular form:

	Half Sleeves	**Full Sleeves**	**Requirement**
Profit (in ₹)	x	$1.50y$	Maximise
M_1	x	$2y$	At most 40
M_2	$2x$	y	At most 40
M_3	$\frac{8}{5}x$	$\frac{8}{5}y$	At most 40

Hence, given L.P.P. is Maximise $Z = x + 1.50y$

subject to constraints

$$x + 2y \le 40, \quad 2x + y \le 40, \quad \frac{8}{5}x + \frac{8}{5}y \le 40, \quad x \ge 0, \quad y \ge 0$$

i.e., $8x + 8y \le 200$

i.e., $x + y \le 25$

We consider the following equations

$x + 2y = 40$

x	0	40
y	20	0

$2x + y = 40$

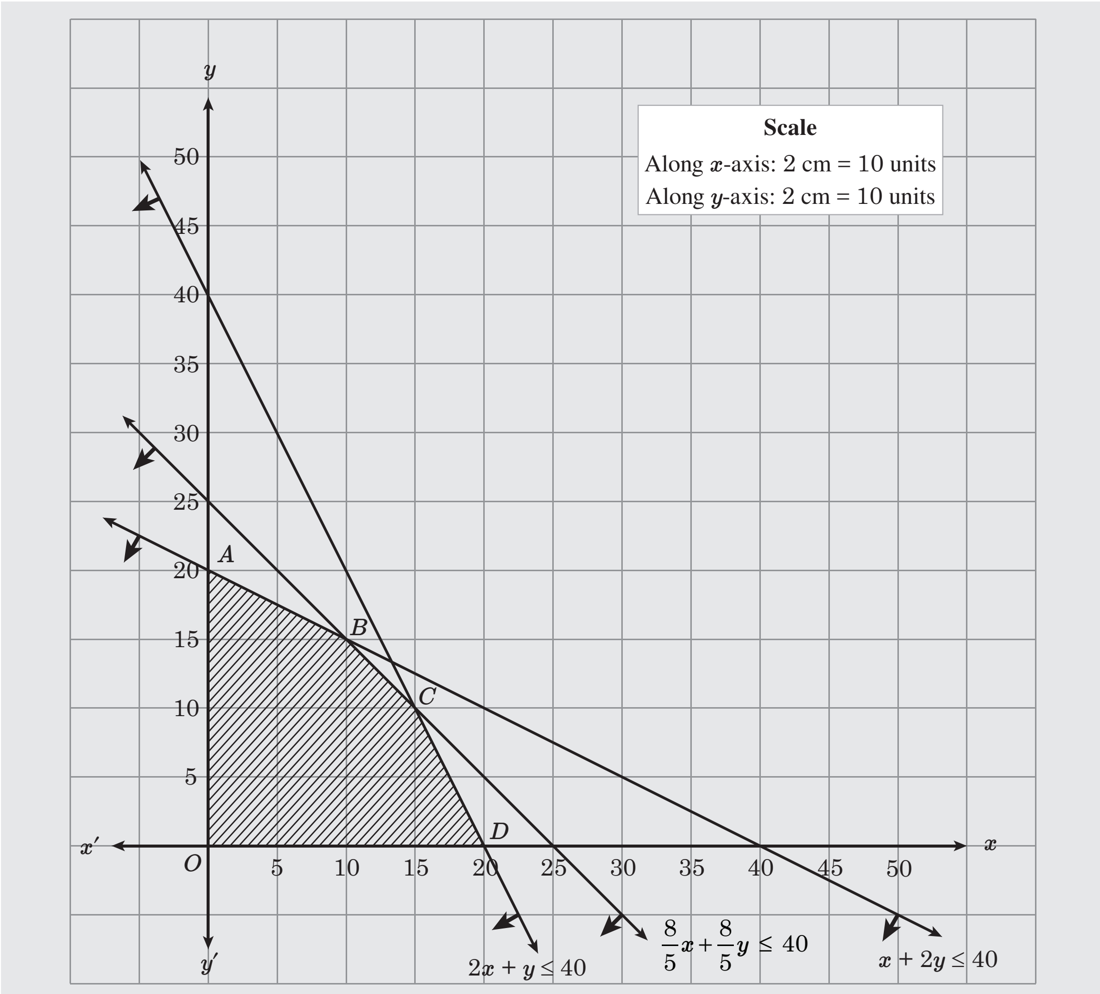
x	0	20
y	40	0

$x + y = 25$

x	0	25
y	25	0

$x = 0, \quad y = 0$

The feasible region is bounded, as shown shaded in the graph.



The values of Z at corner points are as follows:

Corner Points	Value of Z ($Z = x + 1.50y$)
$A(0, 20)$	$Z = 0 + 1.50(20) = 30$
$B(10, 15)$	$Z = 10 + 1.50(15) = 32.5$
$C(15, 10)$	$Z = 15 + 1.50(10) = 30$
$D(20, 0)$	$Z = 20 + 1.50(0) = 20$
$O(0, 0)$	$Z = 0 + 1.50(0) = 0$

Since the feasible region is bounded and 32.5 is the maximum value of Z at corner point.

$\therefore$ 32.5 is maximum value of Z in the feasible region at $x = 10$, $y = 15$.

Hence, the number of half sleeves shirts = 10, number of full sleeves shirts = 15 and maximum profit = ₹ 32.5.