Solutions—Sample Question Paper (Basic) 2024–25

Section A

1. (*b*) HCF = $3^2 \times 5 \times 2 = 90$.

2. (*a*) Given lines are intersecting lines.

Hence, the system of linear equations is consistent with unique solution.

3. (*d*) $kx^2 - 5x + 1 = 0$ does not have a real solution, when D < 0.

i.e., $(-5)^2 - 4k.1 < 0$

or $4k > 25$ or $k > \frac{25}{4}$. Since $7 > \frac{25}{4}$, the given equation does not have a real solution for $k = 7$.

4. (*c*) Distance = $\sqrt{[a-(-a)]^2 + [b-(-b)]^2} = \sqrt{(2a)^2 + (2b)^2} = 2\sqrt{a^2 + b^2}$.

5. (*d*) $\angle QOR + \angle QPR = 180°$ gives

$\angle QOR = 180° - \angle QPR = 180° - 35° = 145°$.

6. (*b*) $\Delta ABC \sim \Delta PQR$ yields

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{\frac{2}{3}PQ}{PQ} = \frac{10}{QR} \Rightarrow \frac{2}{3}QR = 10 \quad \Rightarrow \quad QR = 15 \text{ cm.}$$

7. (*a*) $3 \cot A = 4$ gives $\cot A = \frac{4}{3}$ or $\tan A = \frac{3}{4}$.

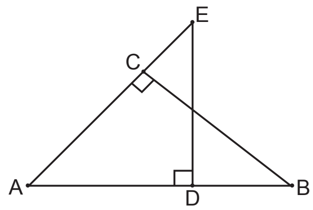
$\therefore \quad \sec^2 A = 1 + \tan^2 A = 1 + \frac{9}{16} = \frac{25}{16}$

$\Rightarrow \quad \sec A = \frac{5}{4}$.

8. (*b*) In ΔBAC and ΔEAD,

$\angle A = \angle A$ and $\angle C = \angle D = 90°$

So, by *AA similarity criterion*, $\Delta BAC \sim \Delta EAD$.



9. (*c*) HCF × LCM = Product of numbers

$\Rightarrow \quad 21 \times \text{LCM} = 420 \times 189$

$\Rightarrow \quad \text{LCM} = \frac{420 \times 189}{21} = 20 \times 189 = 3780.$

10. (*b*) Given A.P. in reverse order is 49, 46, ..., –2, –5, –8

Here, $a = 49$, $d = -3$

4th term from the end means 4th from the beginning of the A.P. in reverse order.

So, $a_4 = a + 3d = 49 - 3 \times 3 = 40$.

11. (*a*) Since $\Delta OCA \sim \Delta OBD$, we have $\angle OAC = \angle ODB = 58°$.

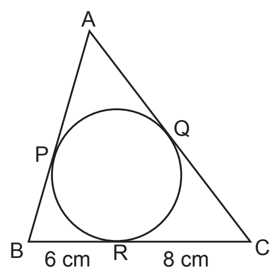
12. (*b*) In the given figure, BP = 6 cm and CQ = 8 cm

Also, (AP + PB) + (AQ + QC) + 14 cm = 38 cm

$\Rightarrow$ (AP + 6) + (AQ + 8) = 24 cm

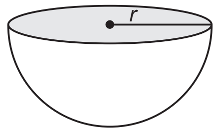
$\Rightarrow$ AP + AQ = 10 cm

$\Rightarrow$ 2 AP = 10 cm or AP = 5 cm.



13. (*a*) $\dfrac{1-\tan^2 30°}{1+\tan^2 30°} = \dfrac{1-\left(\dfrac{1}{\sqrt{3}}\right)^2}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{1-\dfrac{1}{3}}{1+\dfrac{1}{3}} = \dfrac{2}{4} = \dfrac{1}{2} = \cos 60°.$

14. (*c*) Total surface area of a hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$.



15. (*d*) Since $0 \le P(E) \le 1$, P(E) cannot be 4.

16. (*b*) Here, $D = \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

Hence, roots are real and equal.

17. (*c*) Corresponding to the given distribution, we have the following frequency table:

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Students	2	10	16	28	20	4
Cumulative Frequency	2	12	28	56	76	80

$\therefore$ N = 80 and $\dfrac{N}{2} = 40$.

Since the cumulative frequency just greater than 40 is 56, the *median class* is 30–40.

18. (*d*) Quadratic polynomial with given zeros $\dfrac{2}{5}$ and $-\dfrac{1}{5}$ is

x^2 – (Sum of zeros)x + (Product of zeros)

$= x^2 - \dfrac{1}{5}x - \dfrac{2}{25}$

or $25x^2 - 5x - 2$.

19. (*a*) A sequence is said to be an A.P. if the common difference, *i.e.,* $a_n - a_{n-1}$ is a constant. The common difference of the sequence given in Assertion (A) is $-1 - (-1) = 0$, *i.e.,* a constant. So the given sequence is an A.P. Therefore, Assertion (A) is true.

The statement given in Reason (R) is true and the correct explanation of Assertion (A). Therefore, option (*a*) is the correct option.

20. (*c*) $\left(2+\sqrt{3}\right)\sqrt{3} = 2\sqrt{3} + 3$. Since sum of an irrational and a rational number is an irrational number, $\left(2+\sqrt{3}\right)\sqrt{3}$ is an irrational number. Therefore, Assertion (A) is true.

Product of two irrational numbers may be an irrational or a rational number. Therefore, statement given in Reason (R) is false. Thus, option (*c*) is the correct option.

Section B

21. (A) Since P(x, y) is a point equidistant from the points A(4, 3) and B(3, 4), we have

$$AP^2 = BP^2$$

$$(x-4)^2 + (y-3)^2 = (x-3)^2 + (y-4)^2$$

$\Rightarrow$ $x^2 - 8x + 16 + y^2 - 6y + 9 = x^2 - 6x + 9 + y^2 - 8y + 16$

$\Rightarrow$ $2x = 2y$ or $x = y$ or $x - y = 0$.

OR

(B) Since ΔABC is an equilateral triangle, CA = CB = AB = 6

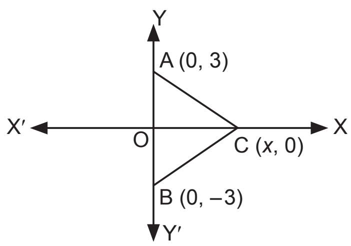
Let the coordinates of point C be (x, 0).

Then, $(x-0)^2 + (0-3)^2 = (x-0)^2 + (0+3)^2 = 36$

$\Rightarrow$ $x^2 + 9 = 36 \Rightarrow x^2 = 27$

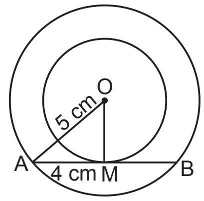
$\Rightarrow$ $x = 3\sqrt{3}$ [Taking only +ve sign]

Thus, the coordinates of point C are $\left(3\sqrt{3}, 0\right)$.



22. In the figure, AB is the chord of the larger circle which touches the smaller circle at M.

Then, OM $\perp$ AB and AM = MB = 4 cm.



From right-angled triangle AMO,

$$OM^2 = OA^2 - AM^2 = 25 - 16 = 9$$

$\Rightarrow$ OM = 3 cm.

Hence, the radius of the smaller circle is 3 cm.

23. (A) Let a and d respectively be the first term and common difference of the A.P. Then,

$$a = 20 \text{ and } S_{12} = 900$$

Therefore, $S_{12} = 900$

$\Rightarrow$ $\frac{12}{2}[2 \times 20 + 11d] = 900$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ $40 + 11d = 150$

$\Rightarrow$ $11d = 110$ or $d = 10$

Now, 12th term, $a_{12} = a + 11d$ $[\because a_n = a + (n-1)d]$

$= 20 + 11 \times 10 = 20 + 110 = 130$

Thus, common difference of the A.P. is 10 and 12th term is 130.

OR

(B) Given, $S_n = 6n - n^2$

Putting $n = 1$ and $n = 2$, we get

$S_1 = 6 - 1 = 5$ and $S_2 = 12 - 4 = 8$

$\Rightarrow$ $a = 5$ and $2a + d = 8$ $[\because S_1 = a \text{ and } S_2 = 2a + d]$

$\Rightarrow$ $d = -2$

Thus, the common difference of the given A.P. is –2.

24. Given, $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, we have

$A - B = 30°$ $[\because \sin 30° = 1/2]$

and $A + B = 60°$ $[\because \cos 60° = 1/2]$

$\Rightarrow$ $A = 45°$ and $B = 15°$.

25. In the given frequency distribution *modal class* is 15–20.

Corresponding to this class, $l = 15$, $f_1 = 15$, $f_0 = 6$, $f_2 = 10$ and $h = 5$.

So, $$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 15 + \frac{15 - 6}{30 - 6 - 10} \times 5 = 15 + \frac{9}{14} \times 5 = 18.21 \text{ (approx).}$$

Thus, mode of the given distribution is 18.21 (approx).

Section C

26. Let us assume, to the contrary, that $\sqrt{5}$ is a rational number.

Then, $\sqrt{5} = \frac{p}{q}$, where p and q are integers having no common factor other than 1 and $q > 0$.

$\Rightarrow$ $5 = \frac{p^2}{q^2}$

$\Rightarrow$ $p^2 = 5q^2$

$\Rightarrow$ 5 divides p^2 $\Rightarrow$ 5 divides p $[\because 5 \text{ is prime}]$

Let $p = 5m$ for some integer m.

Then, $5q^2 = p^2 = 25m^2$ $[\because p^2 = 5q^2]$

$\Rightarrow$ $q^2 = 5m^2$

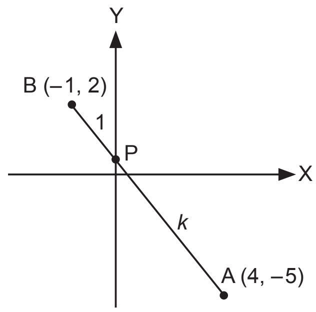
$\Rightarrow$ 5 divides q^2

$\Rightarrow$ 5 divides q

Thus, both p and q have a common factor 5, which is a contradiction to our assumption.

Hence, $\sqrt{5}$ is an irrational number.

27. (A) Let y-axis divides the line segment joining the points A(4, –5) and B(–1, 2) in the ratio $k : 1$.



Then, by section formula,

$$P\left(\frac{-1\times k+4\times 1}{k+1}, \frac{2\times k+(-5)\times 1}{k+1}\right),\ i.e.,\ P\left(\frac{-k+4}{k+1}, \frac{2k-5}{k+1}\right)$$

Since P lies on y-axis, we have

$$\frac{-k+4}{k+1} = 0$$

$\Rightarrow$ $k = 4$

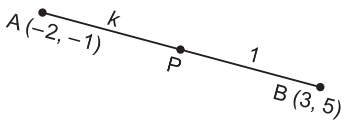
Therefore, required ratio is 4 : 1.

Hence, $\dfrac{2k-5}{k+1} = \dfrac{2\times 4-5}{4+1} = \dfrac{3}{5}$

Thus, the point of intersection P is $\left(0, \dfrac{3}{5}\right)$.

OR

(B) Let the required ratio be $k : 1$.



Then, by section formula,

$$P\left(\frac{k\times 3+1\times(-2)}{k+1}, \frac{k\times 5+1\times(-1)}{k+1}\right)\ i.e.,\ P\left(\frac{3k-2}{k+1}, \frac{5k-1}{k+1}\right)$$

As P lies on line $4x + y = 4$, we have

$$4\left(\frac{3k-2}{k+1}\right)+\left(\frac{5k-1}{k+1}\right) = 4$$

$\Rightarrow$ $12k - 8 + 5k - 1 = 4k + 4$

$\Rightarrow$ $13k = 13 \Rightarrow k = 1$

Hence, the required ratio is 1 : 1.

28. L.H.S. $= (\text{cosec A} - \sin \text{A})(\sec \text{A} - \cos \text{A})$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A}\cdot\frac{\sin^2 A}{\cos A} = \sin A \cos A \quad ...(1)$$

$$\text{R.H.S.} = \frac{1}{\tan A+\cot A} = \frac{1}{\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}} = \frac{\sin A\cos A}{\sin^2 A+\cos^2 A} = \sin A\cos A \quad ...(2)$$

From (1) and (2), we have

$$(\text{cosec A} - \sin A)(\sec A - \cos A) = \frac{1}{\tan A+\cot A}$$ **Proved.**

29. Let the assumed mean be 25, *i.e.,* $A = 25$.

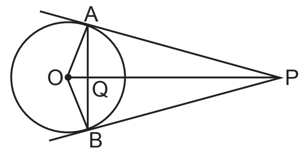
Class	*Mid-point* (x)	*Frequency* (f)	*Deviation* $\left(u = \frac{x-25}{10}\right)$	*Product* (fu)
0–10	5	6	–2	–12
10–20	15	10	–1	–10
20–30	25	15	0	0
30–40	35	9	1	9
40–50	45	10	2	20
Total		$\sum f = 50$		$\sum fu = 7$

$$\therefore \quad \text{Mean} = A + \frac{\Sigma fu}{\Sigma f} \times h$$

$$= 25 + \frac{7}{50} \times 10 = 25 + 1.4 = 26.4$$

Thus, mean of the given distribution is 26.4.

30. (A) (*i*) Consider Δs OAP and OBP



Here, PA = PB, [Tangents from A]

OA = OB [Radii of same circle]

and OP = OP [Common]

$\therefore$ $\Delta OAP \cong \Delta OBP$ [By *SSS congruence criterion*]

$\Rightarrow$ $\angle APO = \angle BPO$

or OP bisects $\angle APB$.

(*ii*) Consider Δs AQP and BQP

Here, PA = PB,

$\angle APQ = \angle BPQ$ [$\angle APO = \angle BPO$]

and QP = QP [Common]

$\therefore$ $\Delta AQP \cong \Delta BQP$ [By *SAS congruence criterion*]

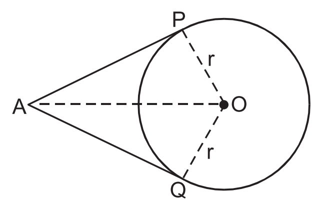
$\Rightarrow$ AQ = BQ

and $\angle AQP = \angle BQP = 90°$ [$\because$ AB is a straight line]

Hence, OP is the right bisector of AB.

OR

(B) Given: AP and AQ are two tangents from A to a circle C(O, r).



To prove: AP = AQ

Construction: Draw the line segments OA, OP and OQ.

Proof: $\angle OPA = \angle OQA = 90°$

[Tangent is $\perp$ to the radius through the point of contact]

In right-angled Δs OPA and OQA, we have

OP = OQ = r [Radius]

OA = OA [Common]

$\therefore$ $\Delta OPA \cong \Delta OQA$ [By *RHS congruence criterion*]

Consequently, AP = AQ.

31. Let the required 2-digit number be $10a + b$. Then,

According to condition, $a = b + 3$

and $(10a + b) + (10b + a) = 99$

$\Rightarrow (10b + 30 + b) + (10b + b + 3) = 99$

$\Rightarrow 22b + 33 = 99$

$\Rightarrow b = 3$

Hence, $a = 3 + 3 = 6$

So, the required 2-digit number is $10 \times 6 + 3$, *i.e.,* 63.

Section D

32. (A) Let the initial price of one book be ₹ p and Amita purchased 'n' books. Then,

$$p.n = 1920 \quad ...(1)$$

and $(n + 4)(p - 24) = 1920$

$\Rightarrow np + 4p - 24n - 96 = 1920$

$\Rightarrow 4p - 24n - 96 = 0$ or $p - 6n - 24 = 0$

$\Rightarrow \dfrac{1920}{n} - 6n - 24 = 0$ $\quad\left[\because p = \dfrac{1920}{n}, \text{ from (1)}\right]$

$\Rightarrow 6n^2 + 24n - 1920 = 0$ or $n^2 + 4n - 320 = 0$

$\Rightarrow (n + 20)(n - 16) = 0 \Rightarrow n = 16$

So, $p = \dfrac{1920}{n} = 120$

Thus, Amita bought 16 books at a price of ₹ 120 each.

OR

(B) Let the average speed of the train be x km/h. Then,

According to condition, $\dfrac{132}{x} + \dfrac{140}{x+4} = 4$

$\Rightarrow \dfrac{33}{x} + \dfrac{35}{x+4} = 1$

$\Rightarrow x(x + 4) - 33(x + 4) - 35x = 0$

$\Rightarrow x^2 + 4x - 33x - 132 - 35x = 0$

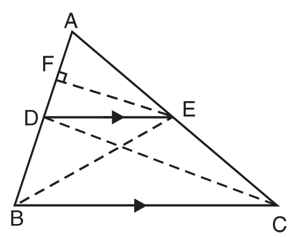
$\Rightarrow x^2 - 64x - 132 = 0$

$\Rightarrow (x - 66)(x + 2) = 0$

$\Rightarrow x = 66$

Thus, the initial average speed of the train is 66 km/h; And the train took $\dfrac{132}{66}$ hours, *i.e.,* 2 hours to cover the distance of 132 km; and $\dfrac{140}{66+4}$ hours, *i.e.,* 2 hours to cover the distance of 140 km.

33. Given: ΔABC in which DE || BC, and intersects AB at D and AC at E.



To prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction: Join BE, CD and draw EF $\perp$ BA.

Proof: Consider the ratio $\dfrac{ar(\Delta\ ADE)}{ar(\Delta\ BDE)}$

We have $\dfrac{ar(\Delta\text{ ADE})}{ar(\Delta\text{ BDE})} = \dfrac{\frac{1}{2}\times \text{AD}\times \text{EF}}{\frac{1}{2}\times \text{DB}\times \text{EF}} = \dfrac{\text{AD}}{\text{DB}}$ ...(1)

Similarly, $\dfrac{ar(\Delta\text{ ADE})}{ar(\Delta\text{ CDE})} = \dfrac{\text{AE}}{\text{EC}}$ ...(2)

But $ar(\Delta\text{BDE}) = ar(\Delta\text{CDE})$

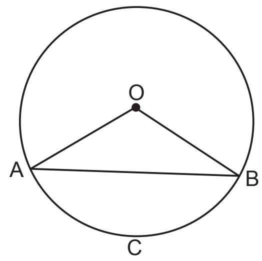
[$\because$ ΔBDE and ΔCDE are on the same base DE and between the same parallel lines DE and BC.]

$\therefore$ L.H.S. of (1) = L.H.S. of (2)

$\Rightarrow$ R.H.S. of (1) = R.H.S. of (2)

$\Rightarrow$ $\dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AE}}{\text{EC}}$ **Proved**.

34. (*i*) Let the central angle AOB be θ. Then,



$$\text{Length of arc ACB} = \frac{\theta}{360}\times 2\pi\,(24)$$

and OA = OB = 24 cm

$\therefore$ Perimeter of sector OACB = OA + OB + arc ACB = 73.12

$\Rightarrow$ $24 + 24 + \dfrac{\theta}{360} \times 2 \times 3.14 \times 24 = 73.12$

$\Rightarrow$ $\dfrac{150.72}{360}\times\theta = 25.12$ $\Rightarrow$ $\theta = 60°$.

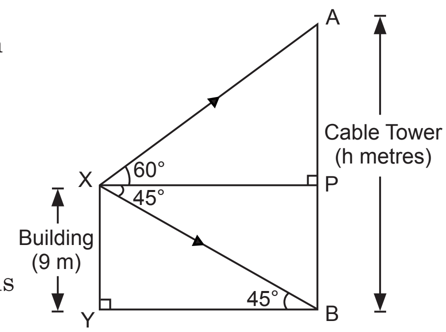
(*ii*) Area of minor segment ACB

= Area of sector OACB – area of ΔOAB

$= \dfrac{60}{360}\times 3.14\times(24)^2 - \dfrac{\sqrt{3}}{4}(24)^2$ [$\because$ ΔOAB is equilateral]

= (301.44 – 249.12) sq cm = 52.32 sq cm.

35. (A) In the figure, XY is 9 m high building and AB is a h m high cable tower. From right triangle XYB, we have



$$\frac{\text{XY}}{\text{YB}} = \tan 45° = 1$$

$\Rightarrow$ XY = YB = 9 m

Thus, the distance between the building and the tower is 9 metres.

Further, from right triangle XPA,

$$\frac{\text{PA}}{\text{XP}} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ $\dfrac{h-9}{9} = \sqrt{3}$ [$\because$ PA = AB – PB = h – 9]

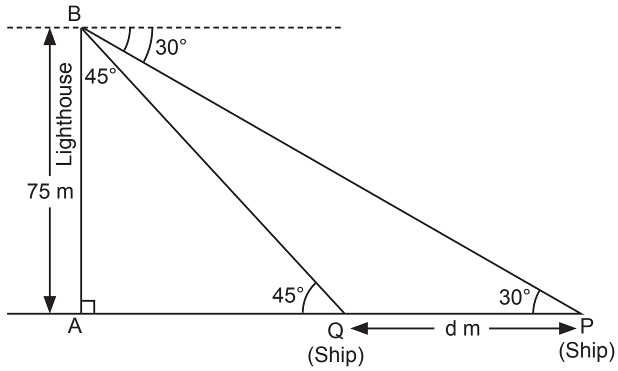
$\Rightarrow$ $\dfrac{h-9}{9} = 1.732$

$\Rightarrow$ $h = 9 + 9 \times 1.732 = 24.588$ m

Thus, the height of the tower is 24.588 metres.

OR

(B) In the figure, AB is the 75 m high lighthouse and P and Q are the positions of the two ships on the same side of the light house. Let the distance between two ships P and Q be 'd' metres.



Then, from right triangle BAQ, we have

$$\frac{AB}{AQ} = \tan 45° = 1$$

$\Rightarrow$ $AQ = AB = 75$ m

Also, from right triangle BAP, we have

$$\frac{AB}{AP} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $AP = \sqrt{3} \times 75 = 129.9$ m

From the figure, we have

$$AP = AQ + d$$

$\Rightarrow$ $d = AP - AQ = 129.9 - 75 = 54.9$ m

Thus, the distance between the two ships is 54.9 metres.

Section E

36. According to the surveyed data,

Number of students who preferred to walk to school = 120

Number of students who preferred to use bicycles = 25% of 200 = 50

Number of students who preferred to take the bus = 10% of 200 = 20

Number of students to be dropped off by car = 200 – (120 + 50 + 20) = 10

$\therefore$ P(student preferred to walk to school) $= \frac{120}{200} = \frac{3}{5}$

P(student preferred to use bicycle) $= \frac{50}{200} = \frac{1}{4}$

P(student preferred to take the bus) $= \frac{20}{200} = \frac{1}{10}$

and P(student preferred to be dropped off by car) $= \frac{10}{200} = \frac{1}{20}$

Thus,

(*i*) P(student does not prefer to walk to school) $= 1 - \frac{3}{5} = \frac{2}{5}$

(*ii*) P(student prefers to walk or use a bicycle) $= \frac{3}{5} + \frac{1}{4} = \frac{17}{20}$

(*iii*) (A) 50% of walking students = 50% of 120 = 60

$\therefore$ Required probability = P(50% of walking students) + P(students comes to school using bicycle)

$$= \frac{60}{200} + \frac{50}{200} = \frac{60+50}{200} = \frac{11}{20}$$

OR

(B) P(students prefers to be dropped off by car) $= \frac{1}{20}$.

37. (*i*) Let α and β be the zeros of the polynomial

$$p(x) = -x^2 + 5x - 4$$

Then, $\alpha + \beta = 5$ ...(1)

and $\alpha\beta = 4$

$\therefore$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= 25 - 16 = 9$

$\Rightarrow$ $\alpha - \beta = 3$ ...(2)

$\therefore$ From (1) and (2), $\alpha = 4$ and $\beta = 1$

Hence, the zeros of $p(x)$ are 1 and 4.

Alternatively: The parabola representing the water fountain, intersects x-axis at (1, 0) and (4, 0). Therefore, the zeros of $p(x)$ are 1 and 4.

(*ii*) Here, $p(x) = -x^2 + 5x - 4$

$$= -\left(x - \frac{5}{2}\right)^2 + \left(\frac{25}{4} - 4\right) = -\left(x - \frac{5}{2}\right)^2 + \frac{9}{4}$$

Thus, the value of x at which water attains maximum height is $\frac{5}{2}$ m.

Alternatively: The perpendicular drawn from the vertex of the given parabola meets the x-axis at 2.5, *i.e.*, $\frac{5}{2}$ m. So, the water attains maximum height at $\frac{5}{2}$ m.

(*iii*) (A) At $x = \frac{5}{2}$, $p(x) = \frac{9}{4}$, or 2.25

Given that the hight of each fountain rod above water level is 10 cm, *i.e.*, 0.1m.

Therefore, $h = 0.10$ m + 2.25 m = 2.35 m.

OR

(B) Putting $p(x) = 2$ in the given equation of the parabola, we have

$-x^2 + 5x - 4 = 2$

$\Rightarrow$ $x^2 - 5x + 4 = -2$

$\Rightarrow$ $x^2 - 5x + 6 = 0$

$\Rightarrow$ $(x - 2)(x - 3) = 0$ $\Rightarrow$ $x = 2$ or 3

Therefore, the required points are (2, 0) and (3, 0).

38. For the given pencil, we have

Length of cylindrical portion, H = 21 cm

Radius of the base, $r = 0.5$ cm; and

Height of conical portion, $h = 1.2$ cm

(*i*) Slant height of the sharpened or conical part $= l$

$$= \sqrt{h^2 + r^2} = \sqrt{(1.2)^2 + (0.5)^2} = 1.3 \text{ cm.}$$

(*ii*) Curved surface area of the sharpened part $= \pi r l$

$$= \pi\left(\frac{1}{2}\right)(1.3) \text{ sq cm} = 0.65\pi \text{ sq cm}$$

(*iii*) (A) Total surface area of pencil $= 2\pi r\text{H} + \pi r^2 + \pi r l$

$= [2\pi\,(0.5)\,(21) + \pi\,(0.5)^2 + 0.65\,\pi]$ sq cm

$= (21\,\pi + 0.25\,\pi + 0.65\,\pi)$ sq cm

$= 21.9\,\pi$ sq cm.

OR

(B) Required volume $= \pi\,(0.5)^2\,(21 - 8.2)$ cu cm

$= \pi\,(0.25)\,(12.8)$ cu cm $= 3.2\,\pi$ cu cm.

Solutions—Sample Question Paper (Standard) 2024–25

Section A

1. (*d*) The graph meets/cuts x-axis at points $(-6, 0)$ and $(6, 0)$.
So, the zeros of $p(x)$ are -6 and 6.

2. (*b*) The system of equation is inconsistent, if

$$\frac{3}{6} = \frac{-k}{10} \neq \frac{7}{3} \text{ i.e., when } k = -5.$$

3. (*d*) No tangent can be drawn from a point inside a circle. Therefore, statement given in option (*d*) is not true.

4. (*a*) Here, $a_1 = 7(1) - 4 = 3$ and $a_2 = 7(2) - 4 = 10$
So, $d = a_2 - a_1 = 10 - 3 = 7$.

5. (*b*)

$$\text{Volume of cone} = \frac{1}{3}\pi(5)^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi(5)^3$$

Since the two volumes are equal,

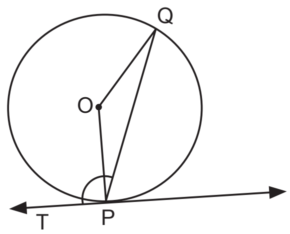
$$\frac{1}{3}\pi(5)^2 h = \frac{4}{3}\pi(5)^3$$

$$\Rightarrow \quad h = 20 \text{ cm.}$$

6. (*a*)

$$\frac{4\sin\theta + \cos\theta}{4\sin\theta - \cos\theta} = \frac{4\tan\theta + 1}{4\tan\theta - 1} \quad \text{[Dividing Nr. and Dr. by } \cos\theta]$$

$$= \frac{4 \times \frac{5}{2} + 1}{4 \times \frac{5}{2} - 1} = \frac{11}{9}.$$



7. (*c*)

$$\angle OPQ = 110° - 90° = 20°$$

$$\Rightarrow \quad \angle OQP = 20°$$

Hence,

$$\angle POQ = 180° - \angle OPQ - \angle OQP$$

$$= 180° - 20° - 20° = 140°.$$

8. (*b*) The quadratic polynomial when its two zeros are given, is

$$x^2 - (\text{Sum of zeros})x + (\text{Product of zeros})$$

$$= x^2 - \left(-\frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2}\right) + \left(-\frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}\right) = x^2 - 0.x - \frac{5}{2}$$

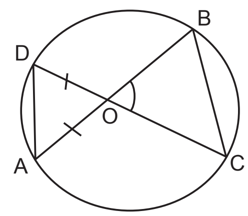
$$= 2x^2 - 5 \quad \text{or} \quad 8x^2 - 20.$$

9. (*c*) The cumulative frequency table of the given distribution is:

Marks	0–10	10–20	20–30	30–40	40–50
Frequency	5	9	15	10	6
Cumulative Frequency	5	14	29	39	45

Here, $N = 45 \Rightarrow \frac{N}{2} = 22.5$.
The cumulative frequency just greater than 22.5 is 29, which lies in the class 20–30.

$\therefore$ Median class is 20–30.
Thus, upper limit of the *median class* is 30.



10. (*b*) Here, $\Delta ODA \sim \Delta OBC$ and $DO.CO = AO.BO$

Since $OA = OD$, two triangles are isosceles and similar.

11. (*a*) Here, discriminant, $D = (1)^2 - 4(1)(-1) = 5 > 0$

So, roots are real and unequal.

Also, $$x = \frac{-1 \pm \sqrt{D}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Thus, roots are irrationals and distinct.

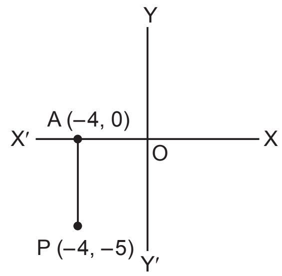
12. (*c*) $3 \tan \theta = 3 \tan 30° = 3 \times \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$.

13. (*b*) Given, $\frac{2}{3}\pi r^3 = \frac{396}{7} \Rightarrow r = 3$

$\therefore$ Total surface area $= 3\pi r^2 = 3 \times \frac{22}{7} \times 9 = \frac{594}{7}$ sq cm.

14. (*b*) Number of white balls $= 24 - 4 - 11 = 24 - 15 = 9$.

So, P(a white ball) $= \frac{9}{24}$, *i.e.*, $\frac{3}{8}$.



15. (*b*) In the figure, $\perp$ drawn from P(–4, –5) meets x-axis at point A(–4, 0) which is nearest to the point (–4, –5).

Thus, the required point is (–4, 0).

16. (*a*) The middle most observation of a data is called the median of the data.

17. (*c*) By section formula, the point P is

$$P\left(\frac{1\times5+2\times2}{1+2}, \frac{1\times6+2\times(-3)}{1+2}\right) = P\left(\frac{5+4}{1+2}, \frac{6-6}{1+2}\right), \text{ i.e., } P(3, 0).$$

18. (*d*) In all there are 12 faces cards in a deck of playing cards, *i.e.*, 6 black and 6 red.

$\therefore$ P(a red face card) $= \frac{6}{52}$ or $\frac{3}{26}$.

19. (*b*) We know that every even natural number is divisible by 2. So, at least 2 will always be a common factor to both of these numbers. Therefore, HCF of two consecutive even natural numbers is always 2. Thus, Assertion (A) is true.

The statement given in Reason (R) is also true but it is not the correct explanation of Assertion (A). Hence, option (*b*) is the correct answer.

20. (*d*) If an arc of a circle of radius r subtends an angle θ at the centre, then

Perimeter of the corresponding sector $= 2r + \frac{\pi r\theta}{180}$

When radius is reduced to half, *i.e.,* $r/2$ and angle is doubled, *i.e.,* 2θ

$$\text{Perimeter of the sector} = 2\left(\frac{r}{2}\right) + \frac{\pi\left(\frac{r}{2}\right)(2\theta)}{180}$$

$$= r + \frac{\pi r\theta}{180}$$

Since the perimeter of the sector is not same, therefore Assertion (A) is false.

The statement given in Reason (R) is a true statement.

Thus, option (*d*) is the correct answer.

Section B

21. (A) By prime factorisation, we have

$$480 = 2^5 \times 3 \times 5 \quad \text{and} \quad 720 = 2^4 \times 3^2 \times 5$$

So, $\text{HCF}(480, 720) = 2^4 \times 3 \times 5 = 240.$

and $\text{LCM}(480, 720) = 2^5 \times 3^2 \times 5 = 1440.$

OR

(B) We have, $85 = 5 \times 17$ and $238 = 2 \times 7 \times 17$

$\therefore$ HCF of 85 and 238 is 17.

So, $17 = 85m - 238$

$\Rightarrow$ $85m = 255 \Rightarrow m = 3.$

22. (A) Total number of possible outcomes when two dice are rolled together = $6 \times 6 = 36$

For a product to be odd, both the numbers in a pair are to be odd.

The numbers on the dice are 4, 6, 7, 9, 11 and 12.

So, favourable outcomes are

(7, 7), (7, 9), (7, 11), (9, 7), (9, 9), (9, 11), (11, 7), (11, 9), (11, 11), *i.e.,* 9 in all.

So, P(product is an odd number) = $\frac{9}{36}$ or $\frac{1}{4}$.

OR

(B) Total number of 3-digit numbers = 900

$\therefore$ Total number of possible outcomes = 900

Numbers with hundredth digit 8 and units digit 5 are 805, 815, 825, 835, ..., 895

$\therefore$ Number of favourable outcomes = 10

So, P(selected one such number) = $\frac{10}{900}$, *i.e.,* $\frac{1}{90}$.

23. We have, $\dfrac{2\sin^2 60^\circ - \tan^2 30^\circ}{\sec^2 45^\circ} = \dfrac{2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{\left(\sqrt{2}\right)^2} = \dfrac{\frac{3}{2} - \frac{1}{3}}{2} = \dfrac{7}{12}.$

24. Let the required point on the x-axis be $(x, 0)$. Then,

$$\sqrt{(x-8)^2 + (0+5)^2} = \sqrt{41}$$

$\Rightarrow$ $(x - 8)^2 + 25 = 41 \Rightarrow (x - 8)^2 = 16$

$\Rightarrow$ $x - 8 = \pm 4 \Rightarrow x = 12$ or $x = 4$

So, the required points are (12, 0) and (4, 0).

25. Here, A(–5, 6), B(3, 0) and C(9, 8) are the given points. Therefore,

$$AB = \sqrt{(3+5)^2 + (0-6)^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = 10$$

$$BC = \sqrt{(9-3)^2 + (8-0)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10$$

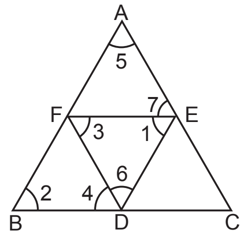
$$CA = \sqrt{(-5-9)^2 + (6-8)^2} = \sqrt{14^2 + 2^2} = \sqrt{196 + 4} = \sqrt{200} = 10\sqrt{2}$$

Since AB = BC, ΔABC is an isosceles triangle.

Section C

26. (A) Since D, E and F are the mid-points of BC, CA and AB respectively.

EF ∥ BC or EF ∥ BD and DE ∥ AB or DE ∥ BF.



⇒ BDEF is a parallelogram.

⇒ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

∴ By *AA similarity criterion*, ΔFBD ~ ΔDEF

Also, DEAF is a parallelogram.

So, $\angle 5 = \angle 6$ and $\angle 3 = \angle 7$

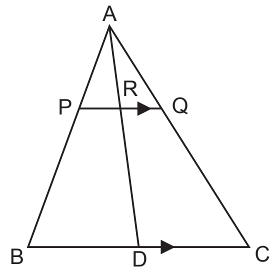
∴ By *AA similarity criterion,* ΔDEF ~ ΔAFE

Further, ΔAFE ~ ΔABC

Hence, ΔDEF ~ ABC.

OR

(B) Given, PQ ∥ BC ⇒ PR ∥ BD,



In Δs APR and ABD

$\angle PAR = \angle BAD$ [Common]

and $\angle APR = \angle ABD$ [Corresponding angles]

∴ By *AA similarity criterion,* ΔAPR ~ ΔABD

$$\Rightarrow \quad \frac{AP}{AB} = \frac{PR}{BD} \qquad ...(1)$$

Similarly, ΔAQR ~ ΔACD

$$\Rightarrow \quad \frac{AQ}{AC} = \frac{RQ}{DC} \qquad ...(2)$$

Also, in ΔABC by *Basic Proportionality Theorem*, we have

$$\frac{AP}{AB} = \frac{AQ}{AC} \qquad ...(3)$$

Using (1), (2) and (3), we get

$$\frac{PR}{BD} = \frac{RQ}{DC}$$

But BD = DC [∵ D is the mid-point of BC]

Thus, PR = RQ or AD bisects PQ.

27. Let the required numbers be x and y. Then,

$$x + y = 18 \qquad ...(1)$$

and
$$\frac{1}{x} + \frac{1}{y} = \frac{9}{40}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{18-x} = \frac{9}{40} \qquad \text{[From (1), } y = 18 - x\text{]}$$

$$\Rightarrow \quad \frac{18}{x(18-x)} = \frac{9}{40}$$

$$\Rightarrow \quad x(18-x) = 80$$

$$\Rightarrow \quad x^2 - 18x + 80 = 0$$

$$\Rightarrow \quad (x-8)(x-10) = 0$$

$$\Rightarrow \quad x = 8 \text{ or } x = 10$$

From (1), when $x = 8$, $y = 10$;

and when $x = 10$, $y = 8$.

Hence, the two numbers are 8 and 10.

28. Given, α and β are the zeros of the polynomial $6x^2 - 5x + 1$. Then,

Sum of zeros, $\alpha + \beta = \frac{5}{6}$ and product of zeros, $\alpha\beta = \frac{1}{6}$

Now, sum of zeros of the required polynomial,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{5}{6}\right)^2 - 2\left(\frac{1}{6}\right) = \frac{25}{36} - \frac{1}{3} = \frac{13}{36}$$

and product of zeros of the required polynomial,

$$\alpha^2\beta^2 = (\alpha\beta)^2 = \frac{1}{36}$$

Thus, the required quadratic polynomial with zeros α^2 and β^2 is

$$x^2 - (\text{Sum of zeros})\, x + \text{Product of zeros} = x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2$$

$$= x^2 - \left(\frac{13}{36}\right)x + \frac{1}{36}$$

or $\quad 36x^2 - 13x + 1.$

29. We have

$$(\cos\theta + \sin\theta)^2 = \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta \quad ...(1)$$

and $\quad (\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta \quad ...(2)$

Adding (1) and (2), we get

$$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2(\cos^2\theta + \sin^2\theta)$$

$\Rightarrow \quad (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2 \quad [\because \cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow \quad 1 + (\cos\theta - \sin\theta)^2 = 2 \quad [\because \cos\theta + \sin\theta = 1]$

$\Rightarrow \quad \cos\theta - \sin\theta = \pm 1.$

30. (A) Angle described by the minute hand in 35 minutes = $7 \times 30° = 210°$

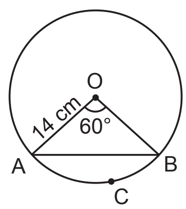
$\therefore$ Area swept by the minute hand in 35 minutes

$$= \pi \times (18)^2 \times \frac{210}{360} \text{ sq cm} \quad \left[\because \text{Area of sector} = \pi r^2 \times \frac{\theta}{360}\right]$$

$= 594$ sq cm.

OR

(B) Area of minor segment = Area of sector OACBO – Area of equilateral ΔAOB



$$= \left[\frac{60}{360} \times \pi \times (14)^2 - \frac{\sqrt{3}}{4}(14)^2\right] \text{ sq cm}$$

$$= \left(\frac{308}{3} - 84.77\right) \text{ sq cm}$$

$= 17.89$ sq cm.

31. Let us assume, to the contrary, that $\sqrt{3}$ is a rational number.

Then $\sqrt{3} = \frac{p}{q}$, where p and q are integers having no common factor, and $q > 0$.

$\Rightarrow \quad 3 = \frac{p^2}{q^2} \quad$ [Squaring both the sides]

$\Rightarrow \quad 3q^2 = p^2$

$\Rightarrow \quad$ 3 divides $p^2 \quad$ [Since 3 divides $3q^2$]

$\Rightarrow$ 3 divides p. [Since 3 is a prime number]

$\Rightarrow$ $p = 3n$, where n is an integer.

Also, $3q^2 = p^2$

$\Rightarrow$ $3q^2 = (3n)^2 = 9n^2$

$\Rightarrow$ $q^2 = 3n^2$

Arguing as above, we get 3 divides q

Thus, both p and q have a common factor 3 which is a contradiction.

Hence, $\sqrt{3}$ is an irrational number.

Section D

32. (A) The given equations are:

$$x + 2y = 3 \qquad ...(1)$$

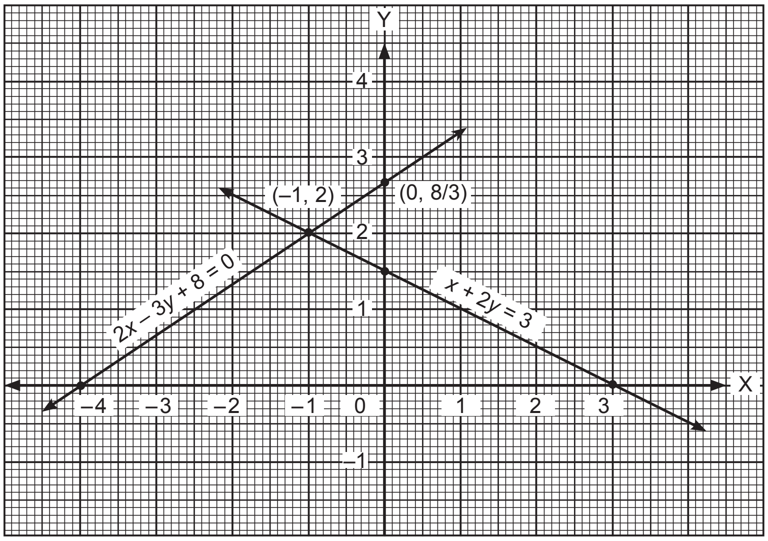
and $$2x - 3y + 8 = 0 \qquad ...(2)$$

Solution table for eq. (1) is:

x	0	3	–1
y	3/2	0	2

Solution table for eq. (2) is:

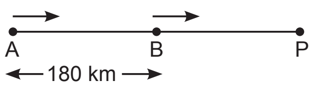
x	0	–4	–1
y	8/3	0	2



Plotting the above points on a graph paper we find that the two lines intersect each other at (–1, 2) which is the required solution of the given system of linear equations.

OR

(B) Let the speed of the two cars be x km/h and y km/h, where $x > y$.



Case 1 : When the two cars are moving in the same direction

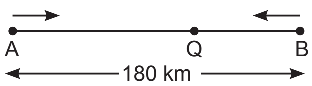
Let the two cars meets at point P. Then,

$$AP - BP = AB$$

$\Rightarrow$ $9x - 9y = 180$

or $x - y = 20$ ...(1)

Case 2 : When the two cars are moving in opposite direction



Let the two cars meets at point Q. Then,

$$AQ + QB = AB$$

$\Rightarrow$ $x + y = 180$ ...(2)

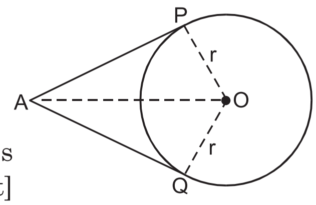
Solving (1) and (2), we get $x = 100$, $y = 80$

Thus, the speeds of the two cars are 100 km/h and 80 km/h.

33. Given: AP and AQ are two tangents from A to a circle C(O, r)

To prove: AP = AQ

Construction: Draw the line segments OA, OP and OQ.



Proof: $\angle OPA = \angle OQA = 90°$ [Tangent is $\perp$ to radius through the point of contact]

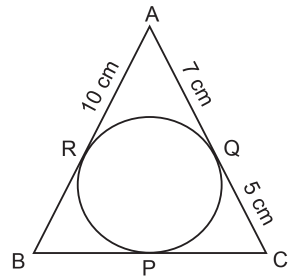
In right-angled Δs OPA and OQA, we have

$OP = OQ = r$ [Radius]

$OA = OA$ [Common]

$\therefore\ \Delta OPA \cong \Delta OQA$ [By *RHS congruence criterion*]

Consequently, AP = AQ **Proved**.



Given, AB = 10 cm, AQ = 7 cm, CQ = 5 cm, we get

CP = CQ = 5 cm, BR = AB – AR = (10 – 7) = 3 cm [$\because$ AR = AQ = 7 cm]

$\therefore$ CP = 5 cm and BP = BR = 3 cm

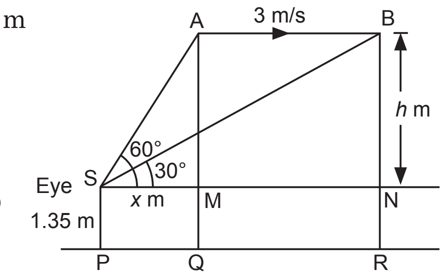
Hence, BC = BP + PC = (3 + 5) = 8 cm.

34. Let S be the boy's eye level and A and B be the two positions of the balloon.

Let h m be the height of the balloon from the ground.

Distances covered by the balloon in 12 seconds = 3 × 12 = 36 metres.

In the figure, AB = QR = 36 m, SP = 1.35 m and SM = x m



From right triangle SMA,

$$\tan 60° = \frac{AM}{SM} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x \qquad ...(1)$$

From right triangle SNB,

$$\tan 30° = \frac{NB}{SN} = \frac{h}{x+36}$$

$$\Rightarrow \quad h = \frac{x+36}{\sqrt{3}} \qquad ...(2)$$

Solving (1) and (2), we have $x = 18$ and $h = 18\sqrt{3}$

$\therefore$ Height of the balloon from the ground, BR = BN + NR

$= \left(18\sqrt{3} + 1.35\right)$ m = (31.14 + 1.35) m = 32.49 m.

35. (A) To compute mean and median, we draw the following table:

Class	*Mid-value* (x)	*Frequency* (f)	*Deviation* $\left(u = \frac{x-102.5}{5}\right)$	*Product* (fu)	*Cumulative Frequency* (cf)
85–90	87.5	15	–3	–45	15
90–95	92.5	22	–2	–44	37
95–100	97.5	20	–1	–20	57
100–105	102.5	18	0	0	75
105–110	107.5	20	1	20	95
110–115	112.5	25	2	50	120
Total		$N = \sum f = 120$		$\sum fu = -39$	

Mean: Let $A = 102.5$ be the assumed mean. Then,

$$\text{Mean} = A + \frac{\Sigma fu}{\Sigma f} \times h$$

$$= 102.5 - \frac{39}{120} \times 5 = 102.5 - 1.625 = 100.875.$$

Median: Here, $\frac{N}{2} = \frac{120}{2} = 60$

Therefore, 100–105 is the *median class*. Corresponding to this class, we have

$$l = 100,\ f = 18,\ cf = 57 \text{ and } h = 5.$$

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h = 100 + \frac{60 - 57}{18} \times 5$$

$$= 100 + \frac{15}{18} = 100.83.$$

OR

(B) First we draw the following table:

Monthly Expenditure (*in* ₹)	*Mid-value* (x)	*Frequency* (f)	*Product* (fx)
1000–1500	1250	24	30,000
1500–2000	1750	40	70,000
2000–2500	2250	33	74,250
2500–3000	2750	$x = 28$	77,000
3000–3500	3250	30	97,500
3500–4000	3750	22	82,500
4000–4500	4250	16	68,000
4500–5000	4750	7	33,250
Total		$\Sigma f = 172 + x$	$\Sigma fx = 5{,}32{,}500$

From the table, we have

$$\Sigma f = 172 + x$$

$$\Rightarrow \quad 172 + x = 200 \Rightarrow x = 28 \qquad [\because \Sigma f = 200]$$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{5{,}32{,}500}{200} = 2662.50.$$

Thus, the value of x is 28 and mean expenditure is ₹ 2662.50.

Section E

36. (*i*) The required A.P. of the jars (from top layer) is 3, 6, 9, ..., 24.

The common difference of the A.P. is 3.

(*ii*) If possible let 34 jars be arranged in nth layer (row). Then,

$$a_n = 34,\ a = 3,\ d = 3$$

So, $3 + (n - 1)(3) = 34 \qquad [\because a_n = a + (n - 1)d]$

$$\Rightarrow \quad n - 1 = \frac{31}{3} \quad \Rightarrow \quad n = \frac{34}{3}, \text{ which is not a +ve integer}$$

Thus, it is not possible to have 34 jars in a layer if the given pattern is continued.

(*iii*) (A) Total number of jars in n rows,

$$S_n = \frac{n}{2}[2 \times 3 + (n-1)(3)]$$
$$= \frac{n}{2}[6 + 3n - 3] = \frac{3n}{2}(n+1)$$

Putting $n = 8$, we get

$$S_8 = \frac{3}{2} \times 8 \times (8+1) = 108.$$

Thus, total number of jars in 8 rows is 108.

OR

(B) On adding 3 jars in each layer, the A.P. is 6, 9, 12, ..., 27.

$\therefore$ Number of jars in 5th layer,

$$a_5 = 6 + 4 \times 3 = 18$$

Hence, there are 18 jars in the 5th layer.

37. (*i*) In the figure, $PQ \parallel EF$

In Δs DPQ and DEF

$$\angle DPQ = \angle DEF$$

and $$\angle DQP = \angle DFE$$

So, by *AA similarity criterion*, $\Delta DPQ \sim \Delta DEF$.

(*ii*) Since $\Delta DPQ \sim \Delta DEF$, we have

$$\frac{DP}{DE} = \frac{PQ}{EF}$$

$$\Rightarrow \quad \frac{50}{50+70} = \frac{PQ}{EF}$$

$$\Rightarrow \quad \frac{50}{120} = \frac{PQ}{EF} \text{ or } \frac{PQ}{EF} = \frac{5}{12}.$$

(*iii*) (A) Given that $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB+BC+AC}{DE+EF+DF}$$

Also, $$2AB = 5DE \quad \Rightarrow \quad AB = \frac{5}{2}DE$$

$$\frac{AB+BC+AC}{DE+EF+DF} = \frac{\frac{5}{2}DE}{DE} = \frac{5}{2}, \text{ which is a constant}$$

Hence, $\dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$ is a constant.

OR

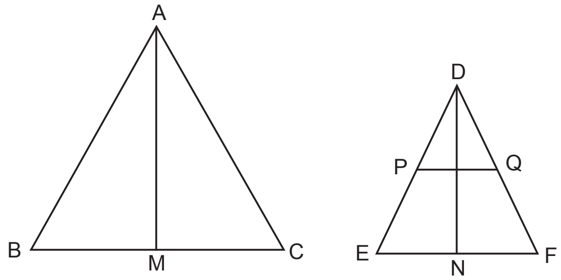
(B) $\Delta ABC \sim \Delta DEF$ gives

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Now, $$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BM}{EN}$$

$[\because BC = 2BM \text{ and } EF = 2EN]$



Thus, in Δs ABM and DEN,

$$\frac{AB}{DE} = \frac{BM}{EN}$$

and $\angle B = \angle E$

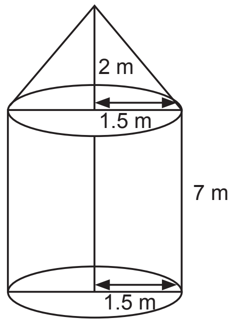
So, by *SAS similarity criterion,* $\Delta ABM \sim \Delta DEN$.

38. For the conical part of one silo, we have

Radius of base (r) = 1.5 m and height (h) = 2 m

For the cylindrical part of one silo, we have

Radius base (r) = 1.5 m and height (H) = 7 m



(*i*) Slant height of the conical part of one silo (l) = $\sqrt{(1.5)^2 + 2^2}$ = 2.5 m.

(*ii*) Curved surface area of the conical part of one silo = πrl

$= \pi(1.5)(2.5)$ sq m

$= 3.75\pi$ sq m

$= 11.78$ sq m.

(*iii*) (A) Cost of metal sheet used to make the curved cylindrical part of 1 silo at the rate of ₹ 2,000 per m^2 = ₹ $(2\pi r\text{H} \times 2000)$

$= ₹ \left(2 \times \frac{22}{7} \times 1.5 \times 7 \times 2000\right)$

= ₹ 1,32,000.

OR

(B) Total capacity of one silo to store grains $= \left(\pi r^2\text{H} + \frac{1}{3}\pi r^2 h\right)$

$= \pi r^2\left(\text{H} + \frac{1}{3}h\right)$

$= \frac{22}{7} \times (1.5)^2\left[7 + \frac{2}{3}\right]$ cu m

$= \frac{22}{7} \times 2.25 \times \frac{23}{3}$ cu m

$= 54.21$ cu m.