Solutions of Question Paper Code: 30/1/1

Section A

Multiple Choice Questions

 $p(x) = x^2 + 3x + k.$ 1. (b) Let Since 2 is one of the zero of p(x), p(2) = 0 $\Rightarrow 2^2 + 3(2) + k = 0$ 4 + 6 + k = 0k = -10 \Rightarrow Factors of a prime number are 1 and the number itself. 2. (c) $x^2 + 5x + 6$ is the polynomial in which sum of zeros is -5 and product of zeros is 6. 3. (a) 4. (d) $\begin{bmatrix} x + y - 4 = 0 \text{ and } 2x + ky = 3 \text{ has no solution, when} \end{bmatrix}$ $\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$ k = 2. \Rightarrow $12 = 2^2 \times 3$ Here, 5. (c) $21 = 3 \times 7$ $15 = 3 \times 5$ So, HCF = 3; LCM = $2^2 \times 3 \times 5 \times 7$, *i.e.*, 420. 6. (a) Since 2x, (x + 10) and (3x + 2) are the three consecutive terms of an A.P., 2(x+10) = 2x + (3x+2)2x + 20 = 5x + 2 \Rightarrow 3x = 18 \Rightarrow x = 6. \Rightarrow Here, a = p and d = q. Then, 7. (c) $a_{10} = a + 9d = p + 9q.$ Distance between the given points 8. (c) $= \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$ $= \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta}$ $=\sqrt{a^2+b^2}$. Here, $P(k, 0) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right), i.e., (-1, 0)$ A(2, -2) P(k, 0) = B(-7, 4)9. (d) k = -1. \Rightarrow 10. (a) Given points are collinear, means 3(p+5) + 5(-5-1) + 7(1-p) = 0i.e., 3p + 15 - 30 + 7 - 7p = 0i.e., 4p = -8p = -2.i.e.,

Fill in the blanks.

11. 10
Here, BC = BQ + QC
= BP + CR (: BQ = BP, QC = CR)
= 3 + (11 - AR) (: CR = AC - AR)
= 14 - AR
= 14 - AP (: AR = AP)
= 14 - 4 = 10.
12.
$$\frac{1}{9}$$
 $\left[\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}.\right]$
13. $\sqrt{3}a$ Altitude AD = $\sqrt{AB^2 - BD^2}$
= $\sqrt{4a^2 - a^2}$
= $\sqrt{3a^2}$
= $\sqrt{3a}.$
14. 2 $\left[\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \csc 31^\circ \\ = \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos(90^\circ - 31^\circ) \csc 31^\circ \\ = \frac{\sin 10^\circ}{\sin 10^\circ} + \sin 31^\circ \csc 31^\circ \\ = 1 + 1 = 2.$
15. 1 $\left[\sin^2\theta + \frac{1}{1 + \tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1.\right]$
OR
1 $\left[(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = (1 + \tan^2\theta)(1 - \sin^2\theta) \\ = \sec^2\theta \cdot \cos^2\theta = 1.\right]$

Very Short Answer Questions

16. Let h be the length of the rod.

Then, its shadow is $\sqrt{3}h$.

If $\boldsymbol{\theta}$ is the angle of elevation, then

$$\tan \theta = \frac{AB}{AX} = \frac{h}{\sqrt{3}h}$$
$$\tan \theta = \frac{1}{\sqrt{3}}$$
$$\theta = 30^{\circ}$$



 \Rightarrow

 \Rightarrow

Hence, the angle of elevation is 30°.

17. Let the heights of two cones be h and 3h; and their base radii be 3r and r.

Then,

$$V_1: V_2 = \frac{1}{3}\pi (3r)^2(h) : \frac{1}{3}\pi (r)^2(3h)$$

 $= 9: 3$
or
 $= 3: 1$

Hence, ratio of their volumes is 3 : 1.

- 18. Of the 26 letters of English alphabets, 21 letters are consonants. So, required probability = $\frac{21}{26}$
- **19.** All possible outcomes are 1, 2, 3, 4, 5 and 6. Favourable outcomes are 1 and 2. So, required probability = $\frac{2}{6}$, *i.e.*, $\frac{1}{2}$.

OR

Probability of losing a game = 1 - P(winning a game)= 1 - 0.07 = 0.93.

20. The mean of first *n* natural numbers is $\frac{n(n+1)}{2n}$, *i.e.*, $\frac{n+1}{2}$ Equating $\frac{n+1}{2}$ to 15, we get n = 29.

Section B

OR

21. $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ will be in A.P., if $2(a^{2} + b^{2}) = (a - b)^{2} + (a + b)^{2}$ $2(a^{2} + b^{2}) = a^{2} + b^{2} - 2ab + a^{2} + b^{2} + 2ab$ i.e., $= 2(a^2 + b^2)$, which is true

Hence, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

22. Consider Δs BDE and BAC.

Here, DE || AC.

- $\frac{BD}{DA} = \frac{BE}{EC}$ So, Consider Δs BDC and BAP. Here, DC || AP. $\frac{BD}{DA} = \frac{BC}{CP}$ So, From (1) and (2), we have $\frac{BE}{EC} = \frac{BC}{CP}$.
- Join OQ.

 $\angle OPQ = \angle OQP$ In $\triangle POQ$, [:: OP = OQ = radii] $\angle POQ = 180^{\circ} - 2 \angle OPQ$ \Rightarrow Further, TPOQ is a cyclic quadrilateral. $\angle POQ = 180^{\circ} - \angle PTQ$ So, $180^{\circ} - 2 \angle OPQ = 180^{\circ} - \angle PTQ$ \Rightarrow $2 \angle OPQ = \angle PTQ$ or $\angle PTQ = 2 \angle OPQ$. \Rightarrow

23. From the figure,
$$\sin \theta = \frac{AC}{DC} = \frac{1.5}{3} = \frac{1}{2}$$

 $\Rightarrow \qquad \theta = 30^{\circ}$
(i) So, $\tan \theta = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$.
(ii) $\sec \theta + \csc \theta$
 $= \sec 30^{\circ} + \csc 30^{\circ}$
 $= \frac{2}{\sqrt{3}} + 2 = \frac{6 + 2\sqrt{3}}{3}$.



...(1)

...(1)

...(2)







24. Let the radius of the circle be 'r' cm.

Then,

 \Rightarrow

$$\widehat{AB} = \frac{60}{360} \times 2 \times \frac{22}{7} \times r = 22$$
$$r = 21 \text{ cm}$$

Thus, the radius of the circle is 21 cm.

25. All possible outcomes are -3, -2, -1, 0, 1, 2 and 3. Favourable outcomes are -2, -1, 0, 1 and 2. So, required probability = ⁵/₇.

26.	Class	Frequency (f)	Class Mark (x)	Product (fx)
	3-5	5	4	20
	5 - 7	10	6	60
	7 - 9	10	8	80
	9-11	7	10	70
	11-13	8	12	96
	Total	$\Sigma f = 40$		$\Sigma f x = 326$

So, Mean
$$= \frac{\Sigma f x}{\Sigma f} = \frac{326}{40} = 8.15.$$

OR

The maximum frequency is 12 for the class 60–80. So, the modal class is 60–80.

For this class, l = 60, h = 20, $f_1 = 12$, $f_0 = 10$ and $f_2 = 6$.

So, Mode =
$$l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

= $60 + \frac{12 - 10}{24 - 10 - 6} \times 20$
= $60 + \frac{40}{8}$
= $60 + 5 = 65$

Thus, mode is 65.

Section C

27. Let α , β be the zeros of $f(x) = \alpha x^2 + bx + c$. Then,

So, the required polynomial is $cx^2 + bx + a$.

 $\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}.$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = -\frac{b}{c}$ $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{\alpha}} = \frac{a}{c}$

Now,

$$[As x^2 \le 4]$$

0

$$\begin{array}{r} x-2 \\ -x^{2} + x - 1 \overline{\smash{\big)} -x^{3} + 3x^{2} - 3x + 5} \\ -x^{3} + x^{2} - x \\ + - + \\ \hline 2x^{2} - 2x + 5 \\ -x^{2} - 2x + 2 \\ - + - \\ \hline 3 \\ \hline \\ \end{array}$$
Here,
$$\begin{array}{r} \text{Dividend} = -x^{3} + 3x^{2} - 3x + 5 \\ \text{Divisor} = -x^{2} + x - 1 \\ \text{Quotient} = x - 2 \\ \text{Remainder} = 3 \end{array}$$

Now,

 $Divisor \times Quotient + Remainder$

$$= (-x^{2} + x - 1)(x - 2) + 3$$

= $-x^{3} + x^{2} - x + 2x^{2} - 2x + 2 + 3$
= $-x^{3} + 3x^{2} - 3x + 5$
= Dividend

Hence, division algorithm is verified.



Vertices of $\triangle ABC$ are A(-4, 2), B(1, 3) and C(2, 5).

OR

Let α , β , γ be the zeros of $p(x) = x^3 - 3x^2 - 10x + 24$. Take $\alpha = 4$. Then,

$$\alpha + \beta + \gamma = 4 + \beta + \gamma = 3 \implies \beta + \gamma = -1 \qquad \dots (1)$$

 $\alpha\beta\gamma = -24 \text{ or } 4\beta\gamma = -24 \implies \beta\gamma = -6$...(2)

and

From (1) and (2), we get

$$(\beta - \gamma)^2 = (\beta + \gamma)^2 - 4\beta\gamma$$

= $(-1)^2 - 4(-6)$
= $1 + 24 = 25$
 \Rightarrow $\beta - \gamma = \pm 5$
Now, $\beta + \gamma = -1$ and $\beta - \gamma = 5$ give $\beta = 2$ and $\gamma = -3$.
and $\beta + \gamma = -1$ and $\beta - \gamma = -5$ give $\beta = -3$ and $\gamma = 2$.
So, the other two zeros are 2 and -3 .

29. Let the original speed of the aircraft be x km/h.

Time taken for distance of 600 km is $\frac{600}{x}$ hours. Time taken for same distance with speed (x - 200) km/h is $\frac{600}{x - 200}$ hours. As per the question,

$$\left[\because 30 \text{ minutes} = \frac{1}{2} \text{hour} \right]$$

[$\therefore x + 400 = 0$ is rejected.]

Thus, the original speed of the aircraft is 600 km/h.

 $\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$

x(x - 200) = 240000

x - 600 = 0

x = 600

 $600 \left[\frac{x - x + 200}{x(x - 200)} \right] = \frac{1}{2}$

 $x^2 - 200x - 240000 = 0$

(x - 600)(x + 400) = 0

So, the original duration of flight = $\frac{600}{600}$ hours, *i.e.*, 1 hour.

30. Area of
$$\triangle PQR = \frac{1}{2} |-5(-5-5) - 4(5-7) + 4(7+5)|$$
 sq units
= $\frac{1}{2} |50 + 8 + 48|$ sq units
= $\frac{1}{2} \times 106$ sq units
= 53 sq units.

OR

y = -2

 $C\left(\frac{3x+8}{7},\frac{3y+20}{7}\right)$

i.e.,

=

 \Rightarrow

$$\frac{3x+8}{7} = -1 \text{ and } \frac{3y+20}{7} = 2$$

 $C\left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = (-1, 2)$

$$x = -5$$
 and

Thus, coordinates of B are (-5, -2).

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

	Here,	$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}}$	[Given]
	or	$\frac{\mathrm{DB}}{\mathrm{AD}} = \frac{\mathrm{EC}}{\mathrm{AE}}$	\bigwedge
	or	$1 + \frac{\text{DB}}{\text{AD}} = 1 + \frac{\text{EC}}{\text{AE}}$	DALE
	or	$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	B∕С
	or	$\frac{AB}{AD} = \frac{AC}{AE}$ or $\frac{AB}{AC} = \frac{AD}{AE}$	
	and	$\angle A = \angle A$	[Common]
	So, by SAS S	Similarity Criterion, $\triangle ADE \sim \triangle ABC$	
	\Rightarrow	$\angle D = \angle B$ and $\angle E = \angle C$	
	Since $\angle D =$	$\angle E$, we get $\angle B = \angle C$	
	∴ In ∆AB	AB = AC	
	Hence, ΔAB	C is an isosceles triangle.	0
32.	Given: In Δ	ABC, $BC^2 = AB^2 + AC^2$	Ň
	To Prove: 2	$\angle BAC = 90^{\circ}$	A
	Constructi PQ = AB an	on: Construct a \triangle PQR, right-angled at P such that d PR = AC.	
	Proof: Sinc	$e \Delta PQR$ is a right triangle, right-angled at P.	
		$QR^2 = PQ^2 + PR^2$	[By Pythagoras' Theorem]
		$= AB^2 + AC^2$	[By construction]
	But	$BC^2 = AB^2 + AC^2$	[Given]
	So,	$QR^2 = BC^2$, or $QR = BC$	
	Now	, in $\triangle ABC$ and $\triangle PQR$,	
		PQ = AB, $PR = AC$ and $QR = BC$	
	So, ł	by SSS Congruence Criterion, we have $\triangle PQR \cong \triangle ABC$	
	Hen	ce, $\angle P = \angle A$	
	<i>i.e.</i> ,	$\angle BAC = 90^{\circ}.$	(Proved)
33.	It is given th	$\tan \sin \theta + \cos \theta = \sqrt{3}$	
	\Rightarrow	$(\sin\theta + \cos\theta)^2 = 3$	
	\Rightarrow	$1 + 2\sin\theta\cos\theta = 3$	
	\Rightarrow	$\sin\theta\cos\theta = 1$	(1)
	Hence,	$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$	
		$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	
		$=\frac{1}{\sin\theta\cos\theta}$	
		_ 1	[D /1\]
		- 1	[Dy (1)]
		= 1	

34. From the figure, $AO' = \frac{1}{2}AO$ Also, $O'E \parallel OC$ So, $\Delta AO'E \sim \Delta AOC$ $\Rightarrow \qquad \frac{AO'}{AO} = \frac{O'E}{OC} \Rightarrow \frac{O'E}{OC} = \frac{1}{2}$ $\Rightarrow \qquad O'E = 2 \text{ cm.} \qquad [\because OC = 4 \text{ cm}]$ Thus, volume of cone ADE $(V_1) = \frac{1}{3}\pi(2)^2h = \frac{4}{3}\pi h$

and volume of cone ABC (V₂) = $\frac{1}{3}\pi(4)^2(2h) = \frac{32}{3}\pi h$

 \Rightarrow Volume of the frustum DBCE = Volume of cone ABC – Volume of cone ADE

$$= \frac{\pi h}{3} (32 - 4)$$
$$= \frac{28}{3} \pi h$$

Thus, the ratio of volumes of two parts = $\frac{4}{3}\pi h$: $\frac{28}{3}\pi h$

Section D

35. Let 'a' be any positive integer. Then, it is of the form

5p or 5p + 1 or 5p + 2 or 5p + 3 or 5p + 4.Case 1: When a = 5p $\Rightarrow a^2 = 25p^2 = 5(5p^2) = 5q, \text{ where } q = 5p^2$ Case 2: When a = 5p + 1 $\Rightarrow a^2 = 25p^2 + 10p + 1 = 5(5p^2 + 2p) + 1 = 5q + 1, \text{ where } q = 5p^2 + 2p$ Case 3: When a = 5p + 2 $\Rightarrow a^2 = 25p^2 + 20p + 4 = 5(5p^2 + 4p) + 4 = 5q + 4, \text{ where } q = 5p^2 + 4p$ Case 4: When a = 5p + 3 $\Rightarrow a^2 = 25p^2 + 30p + 9 = 5(5p^2 + 6p + 1) + 4 = 5q + 4, \text{ where } q = 5p^2 + 6p + 1$ Case 5: When a = 5p + 4 $\Rightarrow a^2 = 25p^2 + 40p + 16 = 5(5p^2 + 8p + 3) + 1 = 5q + 1, \text{ where } q = 5p^2 + 8p + 3$

Thus, square of any positive integer cannot be of the form 5q + 2 or 5q + 3, for any integer *n*.

OR

Let n, n + 1 and n + 2 be three consecutive positive integers.

Also, we know that a positive integer *n* is of the form 3q, 3q + 1 or 3q + 2.

Case I: When n = 3q

Here n is clearly divisible by 3.

But (n + 1) and (n + 2) are not divisible by 3.

[When (n + 1) is divided by 3, remainder is 1 and when (n + 2) is divided by 3, the remainder is 2.]

Case II: When n = 3q + 1

Here, n + 2 = 3q + 3 = 3(q + 1)

Clearly, n + 2 is divisible by 3.

But n and (n + 1) are not divisible by 3.



Case III: When n = 3q + 2

Here, n + 1 = 3q + 3 = 3(q + 1)Clearly, (n + 1) is divisible by 3. But n and (n + 2) are not divisible by 3.

Hence, one of every three consecutive positive integers is divisible by 3.

36. Let 'a' be the first term and 'd' be the common difference of the A.P. Then,

a - 3d, a - d, a + d, a + 3d are four consecutive terms of the A.P.

As per the question,

 $(a-3d) + (a-d) + (a+d) + (a+3d) = 32 \implies 4a = 32$, or a = 8...(1) $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$ and $\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$ \Rightarrow $15a^2 - 135d^2 = 7a^2 - 7d^2$ \Rightarrow $8a^2 = 128d^2$ \Rightarrow Using (1), we have $8 \times 8^2 = 128d^2$ $d^2 = 4$ or $d = \pm 2$ \Rightarrow Thus, the four numbers are 2, 6, 10 and 14. OR In the given A.P., a = 1 and d = 3. Let A.P. contain 'n' terms. Then, $a_n = x$ a + (n-1)d = x \Rightarrow 1 + 3(n - 1) = x \Rightarrow $n = \frac{x+2}{3}$

Further,

 \Rightarrow

 $S = \frac{n}{2}$ [first term + last term]

$$=\frac{x+2}{3\times 2}(1+x)$$

\Rightarrow	(x+1)(x+2) = 1722
or	$x^2 + 3x - 1720 = 0$
\Rightarrow	$x^2 + 43x - 40x - 1720 = 0$
\Rightarrow	x(x+43) - 40(x+43) = 0
\Rightarrow	(x+43)(x-40) = 0
\Rightarrow	x - 40 = 0
\Rightarrow	x = 40

[$\therefore x + 43 = 0$ is rejected]

Thus, x = 40.

20 cm

10 cm

30 cm

★





40. 'More than type' cumulative frequency distribution is as follows:

Production yield (per hectare)	Cumulative Frequency	
More than or equal to 40	100	
More than or equal to 45	96	
More than or equal to 50	90	
More than or equal to 55	74	
More than or equal to 60	54	
More than or equal to 65	24	
More than or equal to 70	0	



OR

Class	Frequency	Cumulative Frequency]
0-100	2	2	
100-200	5	7	
200-300	x	7+x	
300-400	12	19 + x	
400-500	17	36 + x	
500-600	20	56 + x	- Median Class
600-700	У	56 + x + y	
700-800	9	65 + x + y	
800-900	7	72 + x + y	
900-1000	4	76 + x + y	

It is given that $\Sigma f = N = 100$.

So, 76 + x + y = 100

 \Rightarrow x + y = 24

Also, it is given that median is 525.

So, the median class is 500–600.

For this class, $l = 500, h = 100, \frac{N}{2} = 50, c = 36 + x \text{ and } f = 20$ $Median = l + \frac{\frac{N}{2} - c}{f} \times h$

gives

 $525 = 500 + \frac{50 - (36 + x)}{20} \times 100$ 5(14 - x) = 25 \Rightarrow 14 - x = 5 \Rightarrow

x = 9 \Rightarrow

From (1), y = 15

Thus, x = 9 and y = 15.

...(1)