Solutions of Question Paper Code: 30/2/1

Section A

Multiple Choice Questions

1. **(*b*)** [The prime factorisation of 196 is:

$$196 = 2^2 \times 7^2$$

So, sum of the exponents of prime factors 2 and 7 is 2 + 2, *i.e.*, 4.]

2. **(*c*)** [In Euclid's division lemma, the remainder (r) and the divisor (b) satisfy the relation $0 \leq r < b$.]

3. **(*b*)** [Given polynomial can be rewritten as

$$x^2 - (m + 3)x + mx - m(m + 3)$$
$$= x[x - (m + 3)] + m[x - (m + 3)]$$
$$= [x - (m + 3)][x + m]$$

Hence, the two zeros are $m + 3$ and $-m$.]

4. **(*d*)** [The system of equations will be inconsistent when

$$\frac{1}{5} = \frac{2}{k} \neq \frac{3}{-7}$$

i.e., when $k = 10$.]

5. **(*a*)** [The given quadratic equation is $x^2 - 0.04 = 0$

i.e., $x^2 - (0.2)^2 = 0$

or $(x + 0.2)(x - 0.2) = 0$

So, roots are 0.2 and –0.2.]

6. **(*c*)** [The common difference $= \frac{1-p}{p} - \frac{1}{p} = \frac{-p}{p} = -1$.]

7. **(*b*)** [Here, $a = a$ and $d = 2a$

So, $a_n = a + (n - 1)d = a + 2a(n - 1) = 2an - a = a(2n - 1)$.]

8. **(*a*)** [Let the coordinates of P be $(\lambda, 0)$.

Here, $AP^2 = BP^2$

i.e., $(\lambda + 1)^2 = (\lambda - 5)^2$

or $2\lambda + 1 = -10\lambda + 25$

or $12\lambda = 24$ or $\lambda = 2$

So, P(2, 0) is the required point.]

9. **(*c*)** [The reflection of the point (–3, 5) in x-axis is (–3, –5).]

10. **(–)** [Question is wrong.
[P(6, 2) does not lie on the line segment AB.]]

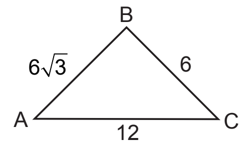
Fill in the blanks.

11. $\frac{1}{9}$ [Here, $\Delta AMN \sim \Delta ABC$

So, $\frac{ar(\Delta AMN)}{ar(\Delta ABC)} = \frac{AM^2}{AB^2} = \frac{1}{9}$ ($\because$ AM : AB = 1 : 3)]

12. 4 [In right ΔAPO, $AP = \sqrt{AO^2 - OP^2} = \sqrt{5^2 - 3^2} = 4$
So, $PB = AP = 4$ cm.]

13. $\angle B = 90°$ [Since $12^2 = (6\sqrt{3})^2 + 6^2$,
$\angle B = 90°$.



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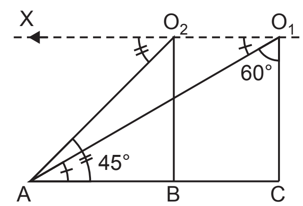
OR

proportional [By definition]

14. 1 [$\tan 1° \tan 2° \ldots \tan 44° \tan 45° \tan 46° \ldots \tan 89°$
$= \tan 1° \tan 2° \ldots \tan 44° (1) \cot 44° \cot 43° \ldots \cot 1°$
$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots (\tan 44° \cot 44°) = 1$.]

15. (30°, 45°) [Angle of depression from O_1 is
$\angle XO_1A = 30°$.
Angle of depression from O_2 is
$\angle XO_2A = 45°$.



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Very Short Answer Questions

16. Here, $\sin A + \sin^2 A = 1$
$\Rightarrow$ $\sin A = 1 - \sin^2 A = \cos^2 A$
Thus, $\cos^2 A + \cos^4 A = \sin A + \sin^2 A$
$= 1$. [Given]

17. Perimeter of the sector AOB = $\overline{OA} + \widehat{AB} + \overline{OB}$

$$= 10.5 + \frac{60}{360} \times 2\pi \times (10.5) + 10.5 = 21 + 11 = 32 \text{ cm.}$$

18. Possible outcome are –3, –2, –1, 0, 1, 2, and 3.
Favourable outcomes are –1, 0 and 1.
So, required probability = $\frac{3}{7}$.

OR

P(a leap year with 52 Sundays) = $\frac{5}{7}$.

[364 days form complete 52 weeks. The remaining two days could be (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon). Out of these 7 choices we have to avoid (Sun, Mon) and (Sat, Sun) to keep 52 Sundays in a leap year.]

19. Class marks are $\frac{10+25}{2}$, *i.e.*, 17.5 and $\frac{35+55}{2}$, *i.e.*, 45.

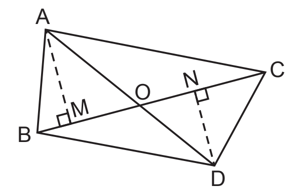
20. Possible outcomes are 1, 2, 3, 4, 5 and 6.
Favourable outcomes are 2, 3 and 5.
So, required probability = $\frac{3}{6}$ or $\frac{1}{2}$.

Section B

21. (*i*) Three, namely $x^3 + \sqrt{3x} + 7$, $2x^2 + 3 - \frac{5}{x}$, $x + \frac{1}{x}$.

(*ii*) One, namely $3x^2 + 7x + 2$.

22. Draw AM ⊥ BC and DN ⊥ BC.



In ΔAMO and ΔDNO, we have

$$\angle AMO = \angle DNO \quad \text{[Each 90°]}$$

and $$\angle AOM = \angle DON \quad \text{[Vert.opp. angles]}$$

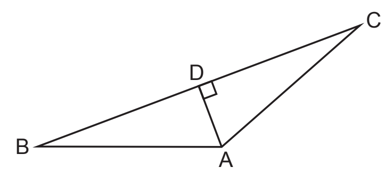
So, by AA Similarity Criterion,

$$\Delta AMO \sim \Delta DNO$$

$$\Rightarrow \quad \frac{AO}{DO} = \frac{AM}{DN} \quad ...(1)$$

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times BC\times DN} = \frac{AM}{DN} = \frac{AO}{DO}. \quad \text{[By (1)]}$$

OR



In right ΔADB,

$$AD^2 = AB^2 - BD^2 \quad ...(1)$$

In right ΔADC,

$$AD^2 = AC^2 - CD^2 \quad ...(2)$$

From (1) and (2),

$$AB^2 - BD^2 = AC^2 - CD^2$$

i.e., $$AB^2 + CD^2 = AC^2 + BD^2.$$

23. L.H.S. $= 1 + \dfrac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}$

$= 1 + \dfrac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$

$= 1 + \dfrac{(\operatorname{cosec}\alpha - 1)(\operatorname{cosec}\alpha + 1)}{1+\operatorname{cosec}\alpha}$

$= 1 + (\operatorname{cosec}\alpha - 1)$

$= \operatorname{cosec}\alpha$ = R.H.S.

OR

We know that

$$\sec^4\theta - \tan^4\theta = (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$$
$$= (\sec^2\theta + \tan^2\theta)[1 + \tan^2\theta - \tan^2\theta]$$
$$= \sec^2\theta + \tan^2\theta$$

$$\Rightarrow \quad \sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta.$$

24. Let the height of the cylinder be 'h' cm. Then,

as per the question, $$V = \pi(h)^2(h) = 25\frac{1}{7} \quad [\because r = h]$$

$$\Rightarrow \quad \frac{22}{7}\cdot h^3 = \frac{176}{7}$$

$$\Rightarrow \quad h^3 = 8$$

or $$h = 2 \text{ cm.}$$

Thus, the height of the cylinder is 2 cm.

25. (*i*) P(getting A) $= \dfrac{2}{6}$, *i.e.*, $\dfrac{1}{3}$.

(*ii*) P(getting D) $= \dfrac{1}{6}$.

26. Here, modal class is 12–16.

For this class,

$$l = 12,\ h = 4,\ f_1 = 17,\ f_0 = 9,\ f_2 = 12$$

So,

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{17-9}{34-9-12} \times 4$$

$$= 12 + \frac{32}{13} = 14.46.\ \text{(approx)}$$

Section C

27. Given equations are

$$2x + y = 23 \quad \text{...(1)}$$

and

$$4x - y = 19 \quad \text{...(2)}$$

Adding (1) and (2), we get

$$6x = 42$$

$$\Rightarrow \quad x = 7$$

Substituting this value of x in (1), we get

$$2 \times 7 + y = 23$$

$$\Rightarrow \quad y = 23 - 14 = 9$$

Thus, $x = 7$ and $y = 9$

So,

$$5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$$

and

$$\frac{y}{x} - 2 = \frac{9}{7} - 2 = -\frac{5}{7}.$$

OR

Given,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \quad (x + 4)(x - 7) + 30 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

$$\Rightarrow \quad (x - 2)(x - 1) = 0$$

i.e.,

$$x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\Rightarrow \quad x = 1 \quad \text{or} \quad x = 2.$$

28. Here, first term is a, and common difference, $d = b - a$.

Let the given, A.P. contain 'n' terms. Then, $a_n = c$

i.e.,

$$a + (n - 1)(b - a) = c \quad \text{or} \quad n = \frac{b+c-2a}{b-a}$$

Now,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{b+c-2a}{2(b-a)}[2a + c - a]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}.$$

OR

Here, $a = 1$ and $d = 3$.

Let $a_n = x$. Then,

$$a + (n-1)d = x \quad ...(1)$$

Now, $$S_n = \frac{n}{2}[2 + 3(n-1)] = 287$$

$\Rightarrow$ $n(3n - 1) = 574$

$\Rightarrow$ $3n^2 - n - 574 = 0$

$\Rightarrow$ $(n - 14)(3n + 41) = 0$

$\Rightarrow$ $n - 14 = 0$ or $3n + 41 = 0$

$\Rightarrow$ $n = 14$ $\left[\because n \neq -\frac{41}{3}\right]$

Thus, from (1), $x = 1 + 13 \times 3 = 1 + 39 = 40$.

29. Let the original speed of the aircraft be x km/h.

Time taken for distance of 600 km is $\frac{600}{x}$ hours.

Time taken for same distance with speed $(x - 200)$ km/h is $\frac{600}{x-200}$ hours.

As per the question,

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2} \quad \left[\because 30 \text{ minutes} = \frac{1}{2}\text{hour}\right]$$

$\Rightarrow$ $600\left[\frac{x - x + 200}{x(x-200)}\right] = \frac{1}{2}$

$\Rightarrow$ $x(x - 200) = 240000$

$\Rightarrow$ $x^2 - 200x - 240000 = 0$

$\Rightarrow$ $(x - 600)(x + 400) = 0$

$\Rightarrow$ $x - 600 = 0$

$\Rightarrow$ $x = 600$ [$\because x + 400 = 0$ is rejected.]

Thus, the original speed of the aircraft is 600 km/h.

So, the original duration of flight = $\frac{600}{600}$ hours, *i.e.*, 1 hour.

30. The mid-point of AB is $\left(\frac{3+k}{2}, \frac{4+6}{2}\right)$, *i.e.*, $\left(\frac{3+k}{2}, 5\right)$

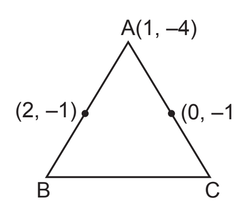
Equating it with (x, y), we have

$$x = \frac{3+k}{2}; \ y = 5$$

Also, $x + y - 10 = 0$ gives $\frac{3+k}{2} + 5 - 10 = 0 \Rightarrow k = 7$.

OR

Here, B(3, 2) and C(–1, 2)

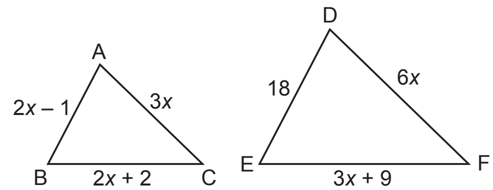


Then, area of $\Delta ABC = \left|\frac{1}{2}[1(2-2) + 3(2+4) - 1(-4-2)]\right|$

$= \left|\frac{1}{2}(0 + 18 + 6)\right|$

$= 12$ sq units.

31. Since $\Delta ABC \sim \Delta DEF$,



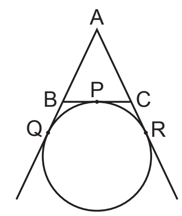
$$\frac{2x-1}{18} = \frac{3x}{6x} = \frac{2x+2}{3x+9}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{1}{2} = \frac{2x+2}{3x+9}$$

$$\Rightarrow \quad x = 5$$

Thus, ΔABC has sides of lengths 9 cm, 15 cm and 12 cm, and ΔDEF has sides of lengths 18 cm, 30 cm and 24 cm.

32. Here, BP = BQ and CP = CR
Also, AQ = AR } ...(1)



Now, AB + BC + CA = (AQ – BP) + (BP + PC) + (AR – CR)
= 2AQ [By (1)]

Hence, $AQ = \frac{1}{2}(AB + BC + CA)$.

33. Given, $\sin\theta + \cos\theta = \sqrt{2}$,

Squaring both sides, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

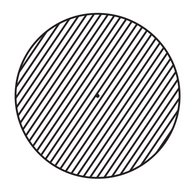
$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2} \quad ...(1)$$

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{2}{1} = 2. \quad \text{[By (1)]}$$

34. Let 'r' cm be the radius of the circular playground. Then,



$$\pi r^2 = 22176$$

$$\Rightarrow \quad r^2 = 22176 \times \frac{7}{22} = 7056$$

$$\Rightarrow \quad r = 84 \text{ cm}$$

$\therefore$ Circumference of the ground $= \left(2 \times \frac{22}{7} \times 84\right)$ cm = 528 cm

and cost of fencing = ₹ $\left(50 \times \frac{528}{100}\right)$ = ₹ 264. $\left[\because 528 \text{ cm} = \frac{528}{100} \text{m}\right]$

Section D

35. Let us assume, to the contrary, that $\sqrt{5}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{5} = \frac{a}{b}$

$$\Rightarrow \quad 5 = \frac{a^2}{b^2}$$

$$\Rightarrow \quad 5b^2 = a^2 \quad ...(1)$$

$\Rightarrow$ a^2 is divisible by 5. [$\because$ $5b^2$ is divisible by 5.]

$\Rightarrow$ a is divisible by 5. [$\because$ 5 is a prime number and divides $a^2 \Rightarrow$ 5 divides a.]

Let $a = 5c$, for some integer c

Substitute $a = 5c$ in (1), we get

$$5b^2 = (5c)^2$$

$$\Rightarrow \quad 5b^2 = 25c^2 \quad \Rightarrow \quad b^2 = 5c^2$$

$\Rightarrow$ b^2 is divisible by 5. [$\because$ $5c^2$ is divisible by 5.]

$\Rightarrow$ b is divisible by 5.

Since a and b both are divisible by 5, 5 is common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ is irrational.

36. Let two pipes A and B of diameters d_1 and d_2 ($d_1 > d_2$) take x and y hours to fill the pool, separately. Then,

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{12} \quad ...(1)$$

and

$$\frac{4}{x}+\frac{9}{y} = \frac{1}{2} \quad ...(2)$$

Multiplying Eq. (1) by 4 and subtracting it from Eq. (2), we get

$$\frac{4}{x}+\frac{9}{y} = \frac{1}{2}$$

$$\frac{4}{x}+\frac{4}{y} = \frac{1}{3}$$

$$- \quad - \qquad -$$

$$\frac{5}{y} = \frac{1}{6} \quad \Rightarrow \quad y = 30$$

Substituting $y = 30$ in Eq. (1), we get

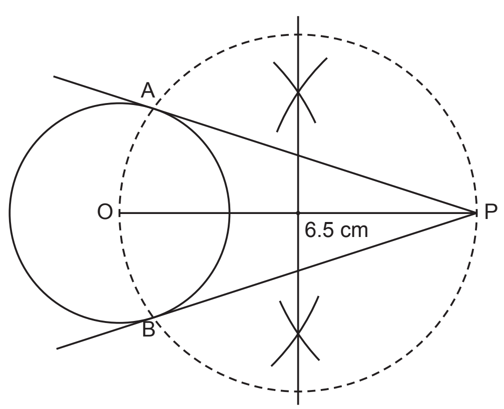
$$\frac{1}{x}+\frac{1}{30} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{12}-\frac{1}{30}$$

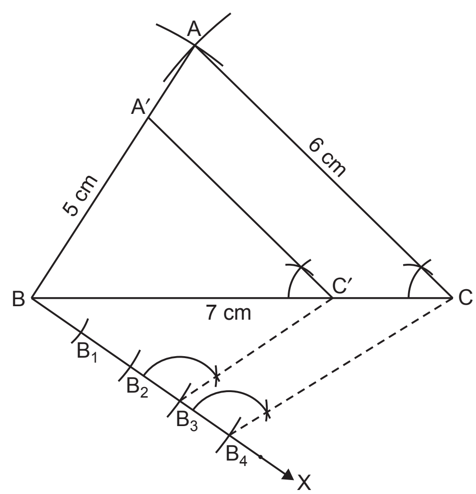
$$\Rightarrow \quad \frac{1}{x} = \frac{1}{20} \quad \text{or} \quad x = 20$$

Thus, the pipe with diameter d_1 takes 20 hours and the pipe with diameter d_2 takes 30 hours to fill the pool alone.

37.



OR

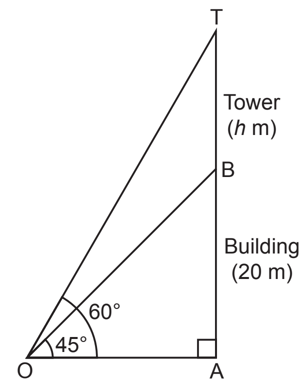


ΔA′BC′ is the required triangle.

38. Let AB and BT represent the building and the tower respectively.

Let O be the position of the observer on the ground.

We need to determine, the height (h) of the tower.



From right ΔOAB, we have

$$\tan 45° = \frac{AB}{OA} \Rightarrow 1 = \frac{20}{OA} \text{ or } OA = 20 \quad ...(1)$$

From right ΔOAT, we have

$$\tan 60° = \frac{AT}{OA} \Rightarrow \sqrt{3} = \frac{20+h}{OA}$$

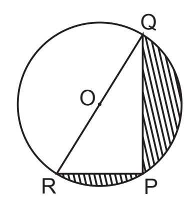
$$\Rightarrow \sqrt{3} = \frac{20+h}{20} \quad \text{[By (1)]}$$

$$\Rightarrow \quad 20 + h = 20\sqrt{3}$$

or $$h = 20\left(\sqrt{3}-1\right) \text{ m.}$$

Thus, the height of the tower is $20\left(\sqrt{3}-1\right)$ metres.

39. In the figure, ΔRPQ is a right triangle where $\angle$P = 90°.



From the figure, we find that

Area of the shaded region = Area of semicircle – Area of ΔRPQ

$$= \frac{1}{2}\pi(OR)^2 - \frac{1}{2} \times RP \times PQ \quad ...(1)$$

In ΔRPQ, $$RQ = \sqrt{RP^2 + PQ^2}$$

$$= \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625} = 25 \text{ cm}$$

$\therefore$ $$OR = \frac{RQ}{2} = 12.5 \text{ cm}$$

Thus, from (1), we have

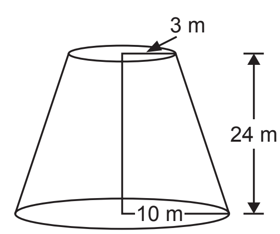
$$\text{Area of the shaded region} = \left[\frac{1}{2} \times \frac{22}{7} \times 12.5 \times 12.5 - \frac{1}{2} \times 7 \times 24\right] \text{ sq cm}$$

$$= [245.53 - 84] \text{ sq cm}$$

$$= 161.5 \text{ sq cm. (approx)}$$

OR

We know that curved surface area of a frustum with circular ends of radii r_1 and r_2 and height h units is given by



$$\pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2}$$

So, required curved surface area $= \frac{22}{7}(10+3)\sqrt{24^2 + 7^2}$

$$= \left(\frac{22}{7} \times 13 \times 25\right)$$

$$= 1021.4 \text{ sq m. (approx)}$$

40.

Class Interval	*Class Mark* (x_i)	*Frequency* (f_i)	*Product* (f_ix_i)
11–13	12	3	36
13–15	14	6	84
15–17	16	9	144
17–19	18	13	234
19–21	20	f	$20f$
21–23	22	5	110
23–25	24	4	96
Total		$\Sigma f_i = 40 + f$	$\Sigma f_ix_i = 704 + 20f$

Here, $\Sigma f_i = 40 + f$ and $\Sigma f_ix_i = 704 + 20f$

We know that $\text{Mean} = \dfrac{\Sigma f_ix_i}{\Sigma f_i}$

$\Rightarrow$ $18 = \dfrac{704 + 20f}{40 + f}$

$\Rightarrow$ $720 + 18f = 704 + 20f$

$\Rightarrow$ $2f = 16$ or $f = 8$.

OR

'More than' type cumulative frequency distribution is:

Production yield (per hectare)	*Cumulative Frequency*
More than or equal to 40	100
More than or equal to 45	96
More than or equal to 50	90
More than or equal to 55	74
More than or equal to 60	54
More than or equal to 65	24
More than or equal to 70	0