Solutions of Question Paper Code: 30/3/1

Section A

Multiple Choice Questions

1. (*c*) [Here, $135 = 3 \times 3 \times 3 \times 5 = 3^3 \times 5^1$
and $225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$
So, HCF(135, 225) $= 3^2 \times 5^1$, *i.e.*, 45.]

2. (*b*) [Here, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$
So, exponent of 2 in the prime factorisation of 144 is 4.]

3. (*a*) [Here, $a_n = 3n + 7$
$\Rightarrow \quad a = a_1 = 3 \times 1 + 7 = 10$
and $a_2 = 3 \times 2 + 7 = 13$
So, $d = a_2 - a_1 = 13 - 10 = 3$.]

4. (*d*) [$x^2 + 4x + \lambda$ is a perfect square when $\lambda = 4$, as $x^2 + 4x + 4 = (x + 2)^2$.]

5. (*b*) [The system of given equations will have infinitely many solutions, when
$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} \quad \Rightarrow \quad k = 1.$$]

6. (*d*) [The terms $(2p + 1)$, 10 and $(5p + 5)$ are consecutive terms of A.P. when
$$2 \times 10 = (2p + 1) + (5p + 5)$$
$\Rightarrow \quad 20 = 7p + 6 \quad \Rightarrow \quad p = 2$.]

OR

(–) [Here, $a = 5$, $d = 4$. Let A.P. contain n terms. Then,
$a_n = 185$ gives
$$a + (n - 1)d = 185$$
$\Rightarrow \quad 5 + 4(n - 1) = 185$
$\Rightarrow \quad 4(n - 1) = 180$
$\Rightarrow \quad n = 46$
But this option is not given in the question.]

7. (*b*) [$x^2 + 4x + \lambda$ is a perfect square when $\lambda = 4$, as $x^2 + 4x + 4 = (x + 2)^2$.]

8. (*b*) [Mid-point of AB $= \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$, *i.e.*, $\left(5+\frac{k}{2}, -1\right)$
Since it is (a, b), we have
$$b = -1,\ 5 + \frac{k}{2} = a$$
From the given relation $a - 2b = 18$, we have $a = 16$
$\Rightarrow \quad 5 + \frac{k}{2} = 16 \quad \Rightarrow \quad k = 22$.]

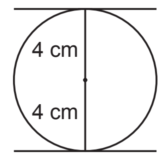
9. (*b*) [A, B and C are collinear, implies
$$0(k + 5) + 2(-5 - 1) + 4(1 - k) = 0$$
i.e., $-12 + 4 - 4k = 0$
$\Rightarrow \quad 4k = -8$
or $k = -2$.]

10. ***(d)*** $\left[\text{Here,} \quad \dfrac{ar(\Delta\text{ABC})}{ar(\Delta\text{DEF})} = \dfrac{\text{AB}^2}{\text{DE}^2} = \dfrac{(1.2)^2}{(1.4)^2} = \dfrac{1.44}{1.96} = \dfrac{36}{49}.\right]$

Fill in the blanks.

11. 10 [Distance between (0, 5) and (–5, 0) = $\sqrt{(-5)^2+(-5)^2} = \sqrt{25+25} = 5\sqrt{2}$
So, $\sqrt{2}$ times the distance = 10.]

12. 8 cm [The distance between two parallel tangents of a circle is equal to the diameter of the circle.



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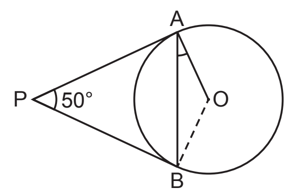
13. 25° [Join OB.
Since APBO is a cyclic quadrilateral,

$$\angle\text{AOB} = 180° - \angle\text{APB}$$
$$= 180° - 50° = 130°$$

Hence, $\angle\text{OAB} = \angle\text{OBA} = \dfrac{1}{2}\,[180° - 130°] = 25°.$



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OR

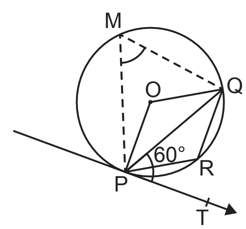
120° [Take a point M on the circle, as shown in the figure.
Join PM and QM.

Here, $\angle\text{PMQ} = \angle\text{QPT} = 60°$

Further, $\angle\text{PRQ} = 180° - \angle\text{PMQ}$

($\because$ PMQR is a cyclic quadrilateral.)

$$= 180° - 60° = 120°.$$



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14. $2\dfrac{1}{2}$

$$\left[\frac{3\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right)\right.$$
$$= \frac{3\cot 40°}{\tan(90° - 40°)} - \frac{1}{2}\left(\frac{\cos 35°}{\sin(90° - 35°)}\right)$$
$$\left.= \frac{3\cot 40°}{\cot 40°} - \frac{1}{2}\left(\frac{\cos 35°}{\cos 35°}\right) = 3 - \frac{1}{2} = 2\frac{1}{2}.\right]$$

15. $\dfrac{49}{64}$

[Here, $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \frac{49}{64}. \qquad \left(\because \cot\theta = \frac{7}{8}\right)\Bigg]$$

Very Short Answer Questions

16. $$\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta} = \frac{1}{\text{cosec}^2\theta} + \frac{1}{\sec^2\theta}$$
$$= \sin^2\theta + \cos^2\theta$$
$$= 1.$$

17. $\text{V}_1 : \text{V}_2 = \dfrac{1}{3}\pi(3r)^2(h) : \dfrac{1}{3}\pi(r)^2(3h)$

$= 9 : 3$ or $3 : 1$.

18. The empirical formula is

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

So,
$$\begin{aligned}\text{Mode} &= 3 \text{ Median} - 2 \text{ Mean}\\ &= 3 \times 8.05 - 2 \times 8.32\\ &= 24.15 - 16.64\\ &= 7.51.\end{aligned}$$

19.
$$\begin{aligned}\text{P(It will not rain tomorrow.)} &= 1 - \text{P(It will rain tomorrow.)}\\ &= 1 - 0.85\\ &= 0.15.\end{aligned}$$

20. Arithmetic mean $= \dfrac{n(n+1)}{2n}$, *i.e.*, $\dfrac{n+1}{2}$.

Section B

21. The given A.P. in the reverse order is

–84, –80, –76, ..., 4, 8, 12

Here, $a = -84, \quad d = 4$

So, 11th term $= a + 10d = -84 + 40 = -44$.

OR

Given, $1 + 5 + 9 + 13 + ... + x = 1326$ ...(1)

Clearly, L.H.S. of (1) is an A.P. whose last term is x. Then,

$$x = 1 + (n-1)(4)$$

$\Rightarrow$ $$x = 4n - 3$$

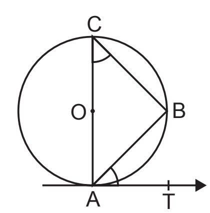
or $$n = \frac{x+3}{4}$$

Further, $$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$ [Using (1)]

$$\begin{aligned}\Rightarrow\quad 1326 &= \frac{x+3}{8}[1+x]\\ \Rightarrow\quad (1+x)(x+3) - 10608 &= 0\\ \Rightarrow\quad x^2 + 4x - 10605 &= 0\\ \Rightarrow\quad (x+105)(x-101) &= 0\\ \Rightarrow\quad x - 101 &= 0 \qquad [\because x + 105 = 0 \text{ is rejected.}]\\ \Rightarrow\quad x &= 101\end{aligned}$$

Thus, the value of x is 101.

22. Here, $\angle B = 90°$ as ΔABC is in the semicircle.



$\therefore$ $\angle BAC + \angle ACB = 90°$ ...(1)

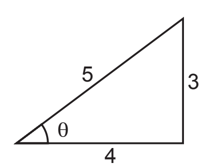
Also, $\angle CAB + \angle BAT = 90°$ ...(2)

[Diameter CA is $\perp$ar to AT.]

From (1) and (2), we have

$$\angle ACB = \angle BAT.$$

23. Given $\tan\theta = \dfrac{3}{4}$, we have $\cos\theta = \dfrac{4}{5}$

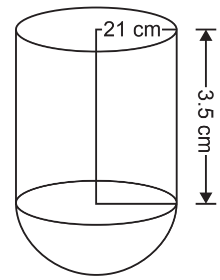


So, $$\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{9}{41}.$$

OR

Given $\tan\theta = \sqrt{3}$, we have $\theta = 60°$

So, $$\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\sec\theta}{\sec^2\theta} = \frac{2}{\sec\theta} = 2\cos\theta = 2 \times \frac{1}{2} = 1.$$



24. (*i*) Curved surface of the top = $2\pi rh$ sq units

$$= 2 \times \frac{22}{7} \times 21 \times 3.5 \text{ sq cm}$$

$$= 462 \text{ sq cm}$$

$\therefore$ Cost of silver coating $= ₹\left(5 \times \frac{462}{100}\right)$

$= ₹\ 23.10.$

(*ii*) Curved surface area of the bottom = $2\pi(r)^2$ sq cm

$$= 2 \times \frac{22}{7} \times 21 \times 21 \text{ sq cm}$$

$$= 2772 \text{ sq cm}$$

Thus, the surface area of glass to be painted red is 2772 sq cm.

25. A leap year has 52 complete weeks + 2 days.

These two days may be

(Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat) and (Sat, Sun).

Of the 7 possible outcomes, only 1, *i.e.*, (Sun, Mon) is the favourable outcome.

So, required probability is $\frac{1}{7}$.

26.

Classes	2–4	4–6	6–8	8–10	10–12	12–14	
Class Mark (x_i)	3	5	7	9	11	13	
Frequency (x_i)	6	8	15	p	8	4	$\Sigma f_i = 41 + p$
Product ($f_i x_i$)	18	40	105	$9p$	88	52	$\Sigma f_i x_i = 303 + 9p$

So, $$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} \text{ gives}$$

$$7.5 = \frac{303 + 9p}{41 + p}$$

$\Rightarrow$ $303 + 9p = 307.5 + 7.5p$

$\Rightarrow$ $1.5p = 4.5$

or $p = 3.$

Section C

27. a, 7, b, 23, c are in A.P. So,

$$7 - a = b - 7 = 23 - b = c - 23$$

From $b - 7 = 23 - b$, we have $b = 15$

From $7 - a = b - 7$, we have $a = -1$ [Using $b = 15$]

From $23 - b = c - 23$, we have $c = 31$ [Using $b = 15$]

$\therefore$ $a = -1$, $b = 15$ and $c = 31$.

OR

Here, $ma_m = m[a + (m - 1)d]$; $na_n = n[a + (n - 1)d]$

Since $ma_m = na_n$ [Given]

$\Rightarrow$ $m[a + (m - 1)d] = n[a + (n - 1)d]$

$\Rightarrow$ $(m - n)a + [m(m - 1) - n(n - 1)]d = 0$

$\Rightarrow$ $(m - n)a + (m^2 - m - n^2 + n)d = 0$

$\Rightarrow$ $(m - n)a + [(m - n)(m + n - 1)]d = 0$ [$m - n \neq 0$ as $m \neq n$]

$\Rightarrow$ $a + (m + n - 1)d = 0$

$\Rightarrow$ $a_{m+n} = 0$

Thus, the $(m + n)$th term of the A.P. is zero.

28. The given quadratic equation will have equal roots, when

$$D = (k + 1)^2 - 4(k + 4)(1) = 0$$

$\Rightarrow$ $(k^2 + 2k + 1) - 4k - 16 = 0$

i.e., $k^2 - 2k - 15 = 0$

i.e., $(k - 5)(k + 3) = 0$

i.e., $k - 5 = 0$ or $k + 3 = 0$

i.e., $k = 5$ or $k = -3$

Thus, the possible values of k are 5 and -3.

29. Let $p(x) = x^3 - 3x^2 + x + 2,$

$r(x) = -2x + 4$

and $q(x) = x - 2$

By division algorithm, we have

$$p(x) = g(x)q(x) + r(x)$$

i.e., $$g(x) = \frac{p(x) - r(x)}{q(x)}$$

$$= \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$= \frac{(x - 2)(x^2 - x + 1)}{x - 2}$$

$$= x^2 - x + 1$$

So, $g(x) = x^2 - x + 1.$

OR

Let α and β be the two zeros of $x^2 - 8x + k$. Then,

$$\alpha + \beta = 8 \text{ and } \alpha\beta = k$$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ gives

$$40 = 8^2 - 2k$$

$\Rightarrow$ $2k = 24$ or $k = 12.$

30. Let P(–4, y) divide AB in the ratio k : 1. Then,

$$(-4, y) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$$

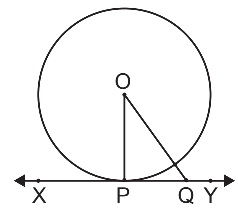
$$\Rightarrow \quad \frac{3k-6}{k+1} = -4 \quad \text{and} \quad y = \frac{-8k+10}{k+1}$$

$$\Rightarrow \quad 3k - 6 = -4k - 4 \quad \text{and} \quad y = \frac{-8k+10}{k+1}$$

$$\Rightarrow \quad k = \frac{2}{7} \quad \text{and} \quad y = \frac{10 - \frac{16}{7}}{\frac{2}{7}+1} = \frac{54}{9} = 6$$

Thus, ratio is 2 : 7 and y = 6.

31. We are given a circle with centre O and a tangent XY to the circle at a point P.



We need to prove that OP ⊥ XY.

Take a point Q on XY, other than P and join OQ.

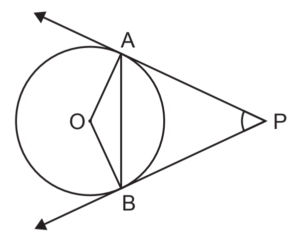
Since Q lies outside the circle, OQ > OP.

As this happens for every point on the line XY, except the point P, OP is the shortest of all the distances of the point O to the points of XY.

So, OP ⊥ XY.

OR

We need to prove that



$$\angle APB + \angle AOB = 180°$$

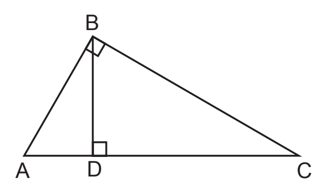
In quad. AOBP, $\angle OAP = 90° = \angle OBP$

[In view of result, proved above]

$$\Rightarrow \quad \angle APB + \angle AOB = 360° - (\angle OAP + \angle OBP)$$
$$= 360° - (90° + 90°)$$
$$= 180°$$

Thus, proved.

32. We are given a right triangle ABC, right-angled at B.



We need to prove that $AC^2 = AB^2 + BC^2$.

Let us draw BD ⊥ AC.

Now, $\Delta ADB \sim \Delta ABC$ [∵ ∠B = ∠D = 90° and ∠A = ∠A (Common)]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$$

or $AD \cdot AC = AB^2$ ...(1)

Similarly, $\Delta BDC \sim \Delta ABC$ [∵ ∠B = ∠D and ∠C = ∠C (Common)]

$$\Rightarrow \quad \frac{CD}{BC} = \frac{BC}{AC}$$

or $CD \cdot AC = BC^2$ ...(2)

Adding (1) and (2), we have

$$AB^2 + BC^2 = AD \cdot AC + CD \cdot AC$$
$$= (AD + CD) \cdot AC$$
$$= AC \cdot AC$$
$$= AC^2$$

Thus, $AC^2 = AB^2 + BC^2$.

33. Given, $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$

Therefore, L.H.S. $= q[p^2 - 1] = (\sec\theta + \text{cosec}\,\theta)[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1]$

$$= 2\sin\theta\cos\theta\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)$$

$$= 2(\sin\theta + \cos\theta)$$

$$= 2p = \text{R.H.S.}$$

34. Amount of water displaced by 500 persons

$$= 0.04 \times 500 \text{ cu m}$$

$$= 20 \text{ cu m}$$

Rise of water level in the pond $= \dfrac{20}{80 \times 50}$ m

$$= 0.005 \text{ m}$$

$$= \frac{1}{2} \text{ cm or 5 mm.}$$

Section D

35. Let us assume that 12^n ends with 0 or 5 for some $n \in \mathrm{N}$.

Then, 5 is a factor of 12^n.

$\therefore \qquad 12^n = 5 \times q$, for some $q \in \mathrm{N}$

$\Rightarrow \qquad (2^2 \times 3)^n = 5 \times q$

$\Rightarrow \qquad 2^{2n} \times 3^n = 5 \times q$

Since the prime number 5 is not a factor of $2^{2n} \times 3^n$, our assumption is wrong.

Hence, 12^n cannot end with 0 or 5 for any $n \in \mathrm{N}$.

OR

Let us assume, on the contrary, that $\sqrt{2} + \sqrt{5}$ is a rational number. Then,

$\sqrt{2} + \sqrt{5} = \dfrac{a}{b}$, where a and b are coprime integers.

$\Rightarrow \qquad \dfrac{a}{b} - \sqrt{2} = \sqrt{5}$

$\Rightarrow \qquad \left(\dfrac{a}{b} - \sqrt{2}\right)^2 = 5$

$\Rightarrow \qquad \dfrac{a^2}{b^2} - 2\sqrt{2}\dfrac{a}{b} + 2 = 5$

$\Rightarrow \qquad \dfrac{a^2}{b^2} - 2\sqrt{2}\dfrac{a}{b} = 3$

$\Rightarrow \qquad \dfrac{a^2}{b^2} - 3 = 2\sqrt{2}\dfrac{a}{b}$

$\Rightarrow \qquad \sqrt{2} = \dfrac{a^2 - 3b^2}{2ab}$

$\Rightarrow \quad \sqrt{2}$ is rational, since $\dfrac{a^2 - 3b^2}{2ab}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is wrong

Hence, $(\sqrt{2} + \sqrt{5})$ is irrational.

36. Let the original uniform speed of the train be 'x' km/h and the length of journey be 'l' km. Then,

Scheduled time taken by the train to cover a distance of 'l' km = $\frac{l}{x}$ hours

Now, as per the question

$$\frac{l}{x+6} = \frac{l}{x} - 4 \quad \text{and} \quad \frac{l}{x-6} - 6 = \frac{l}{x}$$

$$\Rightarrow \quad \frac{l}{x} - \frac{l}{x+6} = 4 \quad \text{and} \quad \frac{l}{x-6} - \frac{l}{x} = 6$$

$$\Rightarrow \quad l\left[\frac{6}{x(x+6)}\right] = 4 \quad \text{and} \quad l\left[\frac{6}{x(x-6)}\right] = 6$$

$$\Rightarrow \quad \frac{4x(x+6)}{6} = \frac{6x(x-6)}{6} \qquad \text{[By eliminating } l\text{]}$$

$$\Rightarrow \quad 2(x+6) = 3(x-6)$$

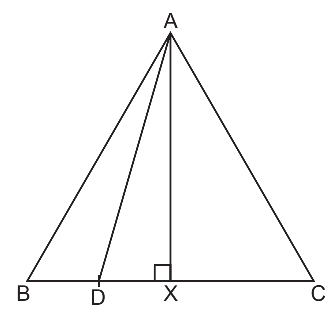
$$\Rightarrow \quad x = 30 \qquad \text{[On cancelling } 2x\text{]}$$

Substituting this value of x in $\frac{l}{x+6} = \frac{l}{x} - 4$, we get

$$l = 720$$

Thus, the length of journey is 720 km.

37. Let each side of ΔABC be '$3a$'.



Then, $\quad BD = a$ and $DC = 2a$

Draw AX $\perp$ BC.

Here, $\quad BX = \frac{3}{2}a = CX$

Now, in ΔAXD,

$$AD^2 = AX^2 + DX^2$$

$$= (AB^2 - BX^2) + (BX - BD)^2$$

$$= \left(9a^2 - \frac{9}{4}a^2\right) + \left(\frac{3}{2}a - a\right)^2$$

$$= \frac{27}{4}a^2 + \frac{a^2}{4}$$

$$= 7a^2$$

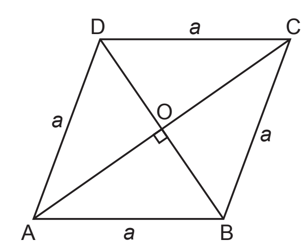
$$= 7\left(\frac{AB}{3}\right)^2 \qquad [\because \ AB = 3a]$$

$$= \frac{7}{9}AB^2$$

$$\Rightarrow \quad 9AD^2 = 7AB^2.$$

OR

Let O be the point of intersection of diagonals AC and BD of the rhombus ABCD.



Let $\quad AC = d_1$ and $BD = d_2$

Here, $\quad AO = CO = \frac{d_1}{2}$; $BO = DO = \frac{d_2}{2}$

Also, $\quad \angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$

Now, in ΔAOB, $\quad AB^2 = AO^2 + BO^2$

$\Rightarrow \qquad AB^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$\Rightarrow \qquad 4AB^2 = d_1^{\,2} + d_2^{\,2}$

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BC^2$ $\qquad [\because AB = BC = CD = DA]$

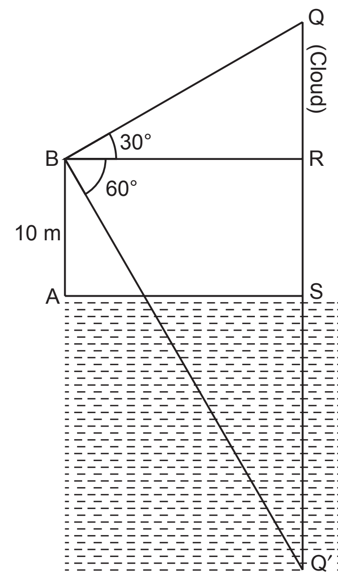
Hence, "sum to the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals".

38. Here, we need to determine the height SQ.

Here, we have SQ = SQ′ and SR = AB = 10 m.

From right ΔBRQ,

$$\frac{QR}{BR} = \tan 30° = \frac{1}{\sqrt{3}} \qquad \text{...(1)}$$



From right $\Delta BRQ'$,

$$\frac{RQ'}{BR} = \tan 60° = \sqrt{3} \qquad \text{...(2)}$$

Eliminating BR from (1) and (2), we have

$$\sqrt{3}QR = \frac{RQ'}{\sqrt{3}}$$

$\Rightarrow \qquad \frac{RQ'}{QR} = 3$

$\Rightarrow \qquad \frac{10 + SQ'}{SQ - 10} = 3 \qquad [\because SQ' = RQ' - 10;\ SQ = 10 + QR]$

$\Rightarrow \qquad \frac{10 + SQ}{SQ - 10} = 3 \qquad [\because SQ' = SQ]$

$\Rightarrow \qquad SQ = 20$ metres

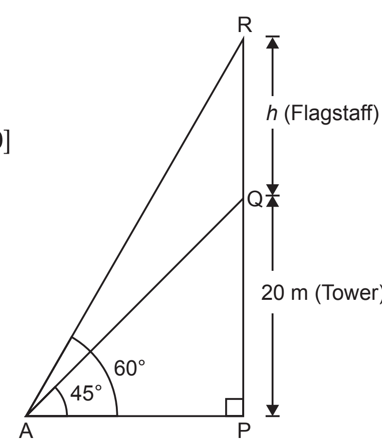
Thus, the height of the cloud from the surface of the lake is 20 metres.

OR

In the figure, PQ is tower and QR is the flagstaff. A is the point of observation.

In right ΔAPQ, we have

$$\frac{PQ}{AP} = \tan 45° = 1$$



$\Rightarrow \qquad AP = 20 \qquad [\because PQ = 20]$

Also, in right ΔAPR, we have

$$\frac{PR}{AP} = \tan 60° = \sqrt{3}$$

$\Rightarrow \qquad AP = \frac{(20 + h)}{\sqrt{3}}$

Eliminating AP, we have

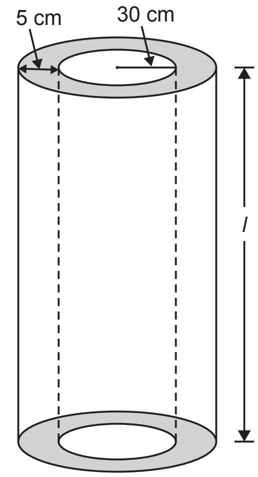
$$\frac{20 + h}{\sqrt{3}} = 20$$

$\Rightarrow \qquad h = 20(\sqrt{3} - 1)$

Hence, the height of flagstaff, *i.e.*, value of h is $20(\sqrt{3} - 1)$ metres.

39. Volume of the cuboidal block = $(4.4 \times 2.6 \times 1)$ cu m

$= 11.44$ cu m or 11440000 cu cm



Let the length of the hollow cylindrical pipe be l cm.

Then, volume of the pipe $= \pi[(35^2) - (30)^2]l$ cu cm

$= 325\pi l$ cu cm

Equating the two volume, we get

$$325\pi l = 11440000$$

$$\Rightarrow \quad l = \frac{11440000}{325} \times \frac{7}{22}$$

$= 11200$ cm or 112 m

Thus, the length of the pipe is 112 m.

40. 'More than' type cumulative frequency distribution is

Class	*Cumulative Frequency*
More than or equal to 0	100
More than or equal to 10	95
More than or equal to 20	80
More than or equal to 30	60
More than or equal to 40	37
More than or equal to 50	20
More than or equal to 60	9
More than or equal to 70	0