Solutions of Question Paper Code: 30/5/1

Section A

Multiple Choice Questions

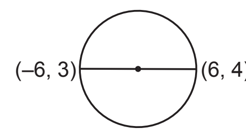
1. **(*b*)** Here, $p(x) = (x^2 - 4)(x) + 3$
$= x^3 - 4x + 3.$

2. **(*a*)** Here, BC = AC
and $AB^2 = AC^2 + BC^2 = 2AC^2$ $(\because AC = BC)$

3. **(*d*)** Since both the given points are on the x-axis, the mid-point $\left(\frac{-4+10}{2}, \frac{0+0}{2}\right)$, *i.e.*, (3, 0) lies on x-axis and is equidistant from (–4, 0) and (10, 0).

OR

(*c*) Since the centre of a circle is the mid-point of its diameter, the centre is $\left(\frac{-6+6}{2}, \frac{3+4}{2}\right)$, *i.e.*, $\left(0, \frac{7}{2}\right)$.

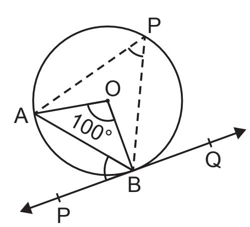


4. **(*b*)** The equation $2x^2 + kx + 2 = 0$ has equal roots, when
$$D = (k)^2 - 4(2)(2) = 0$$
$\Rightarrow$ $k^2 = 16$ or $k = \pm 4.$

5. **(*c*)** Here, $\frac{7}{3} - \frac{4}{3} \neq \frac{9}{3} - \frac{7}{3}.$

6. **(*b*)** For the given pair of equations, we have
$$\frac{\frac{3}{2}}{9} = \frac{\frac{5}{3}}{10} \neq \frac{7}{14}, \text{ or } \frac{1}{6} = \frac{1}{6} \neq \frac{1}{2}$$
Hence, the pair of equations is inconsistent.

7. **(*a*)** Take a point P on the circle as shown in the figure. Join AP and BP and get ΔAPB.

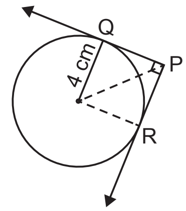


$$\angle ABP = \angle APB$$
$$= \frac{1}{2}\angle AOB$$
$$= \frac{1}{2} \times 100° = 50°.$$

8. **(*c*)** Let 'r' cm be the radius of the sphere. Then,
$$\frac{4}{3}\pi r^3 = 12\pi$$
$\Rightarrow$ $r^3 = 9$
i.e., $r^3 = 3^2$
$\Rightarrow$ $r = (3^2)^{1/3}$, *i.e.*, $3^{2/3}$.

9. **(*c*)** Distance between $(m, -n)$ and $(-m, n)$ is
$$= \sqrt{(-m-m)^2 + (n+n)^2}$$
$$= \sqrt{4m^2 + 4n^2} = 2\sqrt{m^2 + n^2}.$$

10. **(*b*)** [In the figure, OQPR is a square of side 4 cm
So, PQ = 4 cm.]



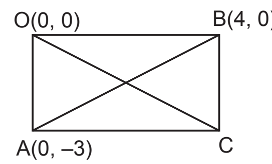
Fill in the blanks.

11. 1 [Probability of a sure event is 1.]

12. $\tan^2 A$ $\left[\dfrac{1+\tan^2 A}{1+\cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A} = \dfrac{1}{\cos^2 A}\cdot\dfrac{\sin^2 A}{1} = \tan^2 A.\right]$

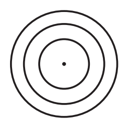
13. 5 units [In a rectangle, both the diagonals are of equal lengths.
So, we shall find AB.

$$AB = \sqrt{(4-0)^2+(0+3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25} = 5 \text{ units.}]$$



14. $\dfrac{x_i - a}{h}$ [In the formula, $\overline{x} = a + \left(\dfrac{\Sigma f_i x_i}{\Sigma f_i}\right) \times h$, $u_i = \dfrac{x_i - a}{h}$.]

15. similar [All concentric circles are similar.]



Very Short Answer Questions

16. The list of first 100 natural numbers

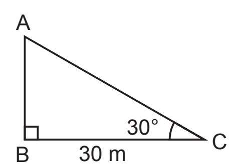
1, 2, 3, ..., 100

forms an A.P. with $a = 1$, $d = 1$

So, $$S_{100} = \frac{100}{2}[2 \times 1 + (100-1)(1)]$$
$$= 50[101]$$
$$= 5050.$$

17. Let the height of the tower BA be h metres.

In right ΔABC, we have



$$\frac{AB}{BC} = \tan 30°$$

$\Rightarrow$ $$\frac{h}{30} = \sqrt{3}$$

$\Rightarrow$ $h = 30\sqrt{3}$ metre = 51.9 m (approx) [Taking $\sqrt{3} = 1.73$]

Hence, the height of the tower is 51.9 metres.

18. We know that

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

So, $$13 \times 182 = 26 \times b$$

$\Rightarrow$ $$b = \frac{13 \times 182}{26} = 91$$

Thus, the other number is 91.

19. A general form of a quadratic polynomial is

$$ax^2 + bx + c$$

Here, $$-\frac{b}{a} = -3 \quad \text{and} \quad \frac{c}{a} = 2$$

Taking $a = 1$, we get $b = 3$ and $c = 2$

So, a required polynomial is $x^2 + 3x + 2$.

OR

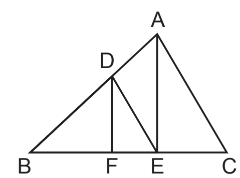
No; in division algorithm,

Remainder = 0 or degree of remainder < degree of divisor.

20. $$\frac{2\tan 45° \times \cos 60°}{\sin 30°} = \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = 2.$$

Section B

21. In ΔABE, DF || AE.



So, by BPT, we have

$$\frac{BD}{DA} = \frac{BF}{FE} \qquad ...(1)$$

In ΔABC, DE || AC.

So, by BPT, we have

$$\frac{BD}{DA} = \frac{BE}{EC} \qquad ...(2)$$

From (1) and (2), we have $\frac{BF}{FE} = \frac{BE}{EC}$.

22. Let us assume, on the contrary, that $5 + 2\sqrt{7}$ is a rational number.

i.e., $$5 + 2\sqrt{7} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are coprimes.}$$

$$\Rightarrow \quad 2\sqrt{7} = \frac{a}{b} - 5$$

$$\Rightarrow \quad \sqrt{7} = \frac{a-5b}{2b}$$

Since $\frac{a-5b}{2b}$ is a rational number, so is $\sqrt{7}$.

This is a contradiction to the given factorisation is an irrational number.

Hence, $5 + 2\sqrt{7}$ is an irrational number.

OR

Let if possible, 12^n have a value which ends with the digit 0

$\Rightarrow$ 10 a factor of 12^n

$\Rightarrow$ 5 is a prime factor of 12^n

i.e., $12^n = 5 \times q$, where q is some natural number.

$\Rightarrow$ $(2^2 \times 3)^n = 5 \times q$

or $2^{2n} \times 3^n = 5 \times q$

The assumption, 5 is a prime factor of $2^{2n} \times 3^n$, is not possible because $2^{2n} \times 3^n$ can have only 2 and 3 as prime factors.

Hence, our assumption is wrong.

Thus, 12^n cannot end with the digit 0.

23. In a ΔABC, $A + B + C = 180°$

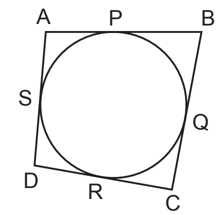
$\Rightarrow$ $B + C = 180° - A$

$\Rightarrow$ $\frac{B+C}{2} = 90° - \frac{A}{2}$

$\Rightarrow$ $\cos\left(\frac{B+C}{2}\right) = \cos\left(90° - \frac{A}{2}\right) = \sin\frac{A}{2}$

Thus, proved.

24. Let quad. ABCD touch the circle at P, Q, R and S as shown in the figure.



Here, $AP = AS; BP = BQ;$

$CQ = CR;$ and $DR = DS$

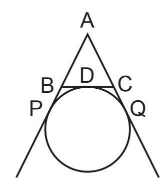
So, $AB + CD = (AP + PB) + (CR + RD)$

$= AS + BQ + CQ + DS$

$= (AS + DS) + (BQ + CQ)$

$= AD + BC.$

OR



Here, BP = BD, CD = CQ and AP = AQ = 12 cm

Now, perimeter of $\Delta ABC = AB + (BD + DC) + AC$

$= (AB + BP) + (CQ + AC)$ [$\because$ BP = BD and CD = CQ]

$= AP + AQ$

$= 2AQ$ [$\because$ AP = AQ]

$= 2 \times 12$

$= 24$ cm.

25. Here, the modal class is 30–40 as this class has the maximum frequency 12.

So, for this class, $l = 30$, $f_1 = 12$, $f_0 = 7$, $f_2 = 5$ and $h = 10$.

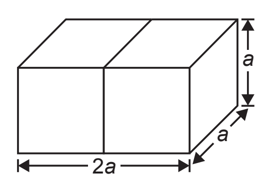
So, $$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{12-7}{24-7-5} \times 10$$

$$= 30 + \frac{50}{12}$$

$$= 34\frac{1}{6}.$$

26. On joining 2 identical cubes, each of edge 'a' cm we get a cuboid of dimensions $2a \times a \times a$.



We are given that $a^3 = 125$, *i.e.*, $a^3 = 5^3$

So, $a = 5$

Hence, the dimensions of the cuboid are $10 \times 5 \times 5$.

So, surface area of the cuboid $= 2[10 \times 5 + 5 \times 5 + 5 \times 10]$ sq cm

$= 2(50 + 25 + 50)$ sq cm

$= 250$ sq cm.

Section C

27. Let the given fraction be $\frac{a}{b}$. Then,

$$\frac{a-1}{b} = \frac{1}{3} \quad \text{and} \quad \frac{a}{b+8} = \frac{1}{4}$$

$$\Rightarrow \quad 3a - b = 3 \quad \text{and} \quad 4a - b = 8$$

Solving these equations, we get

$$a = 5,\ b = 12$$

Thus, the required fraction is $\frac{5}{12}$.

OR

Let 'x' (in years) be the present age of the father and 'y' (in years) be the present age of the son. Then,

$$x = 3y + 3 \quad \text{and} \quad x + 3 = 2(y + 3) + 10$$

$$\Rightarrow \quad x - 3y = 3 \quad \text{and} \quad x - 2y = 13$$

Solving the two equations, we get

$$y = 10, \text{ and } x = 33$$

Thus, father's present age is 33 years and son's present age is 10 years.

28. Let 'a' be any positive integer. Then,

$$a = 3m \quad \text{or} \quad a = 3m + 1 \quad \text{or} \quad a = 3m + 2$$

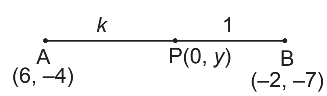
Then, $a^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3q$, where $q = 3m^2$

or $a^2 = (3m + 1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1 = 3q + 1$, where $q = 3m^2 + 2m$

or $a^2 = (3m + 2)^2 = 9m^2 + 12m + 4 = 3(3m^2 + 4m + 1) + 1 = 3q + 1$, where $q = 3m^2 + 4m + 1$

Thus, square of any positive integer is either of the form $3q$ or $3q + 1$, for some integer q.

29. Let P(0, y) divide the line segment AB in the ratio $k : 1$.



Then, $$P(0, y) = \left(\frac{-2k+6}{k+1}, \frac{-7k-4}{k+1}\right)$$

$$\Rightarrow \quad \frac{-2k+6}{k+1} = 0 \quad \text{and} \quad y = \frac{-7k-4}{k+1}$$

$$\Rightarrow \quad k = 3 \quad \text{and} \quad y = \frac{-25}{4}$$

Thus, P divides AB in the ratio 3 : 1 and the point of intersection is $\left(0, \frac{-25}{4}\right)$.

OR

Let the three given points be A(7, 10), B(–2, 5) and C(3, –4).

Then, $AB^2 = (-2 - 7)^2 + (5 - 10)^2 = 81 + 25 = 106$

$BC^2 = (3 + 2)^2 + (-4 - 5)^2 = 25 + 81 = 106$

$CA^2 = (7 - 3)^2 + (10 + 4)^2 = 16 + 196 = 212$

Here, AB = AC and $CA^2 = AB^2 + BC^2$

So, ΔABC is an isosceles right triangle, right-angled at B.

30. L.H.S. $= \sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sqrt{\dfrac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \text{R.H.S.}$$

31. Here, $a = 5$, $d = 3$ and

$$a_n = a + (n-1)d = 50$$

$\Rightarrow \quad 5 + 3(n-1) = 50$

$\Rightarrow \quad 3(n-1) = 45$

$\Rightarrow \quad n - 1 = 15$

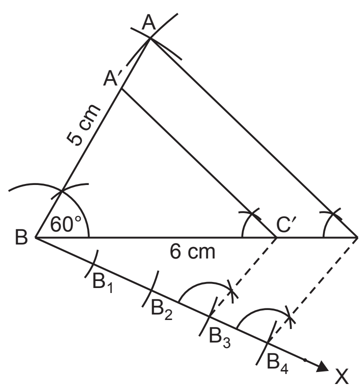
or $\quad n = 16$

Now, $\quad S_n = S_{16} = \dfrac{16}{2}[2 \times 5 + (16-1)(3)]$

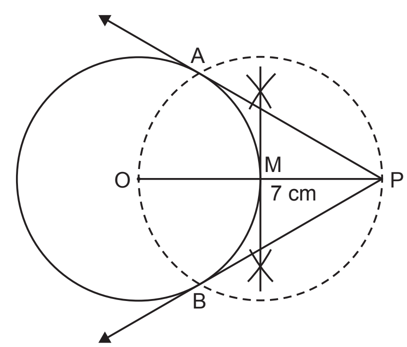
$$= 8[10 + 45] = 440.$$

Hence, the value of n is 16 and sum of first n terms, *i.e.*, 16 terms is 440.

32.



OR



The required triangle is ABC.

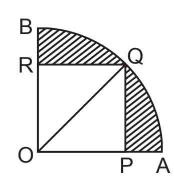
33. (*i*) Out of 6 numbers on the disc, there are 5 even numbers. Shweta will be allowed to pick up a marble, only when the spinner stops on an even number.

$\therefore$ P(getting an even number) $= \dfrac{5}{6}$.

(*ii*) Out of 20 marbles in the bag, there are 6 black marbles.

$\therefore$ P(getting a black marble) $= \dfrac{6}{20}$ or $\dfrac{3}{10}$.

34. In the figure, OPQR is a square.



Here, $\quad OP = PQ$

So, $\quad OQ^2 = OP^2 + PQ^2$

$\Rightarrow \quad 2OP^2 = (6\sqrt{2})^2$

$\Rightarrow \quad OP^2 = 36$

$\Rightarrow \quad OP = 6$ cm

Area of the shaded region = Area of quadrant of the circle of radius $6\sqrt{2}$ cm – Area of the square OPQR of side 6 cm.

$$= \frac{\pi}{4}(6\sqrt{2})^2 - (6)^2$$

$$= 56.57 - 36 = 20.57 \text{ sq cm.}$$

Section D

35. Since $\sqrt{5}$ and $-\sqrt{5}$ are the zeros of $p(x)$, $(x - \sqrt{5})(x + \sqrt{5})$ is a factor of $p(x)$, or $(x^2 - 5)$ is a factor of $p(x)$.

The other two zeros of $p(x)$, we shall get from the quotient on dividing $p(x)$ by $(x^2 - 5)$.

$$\begin{array}{r|l}
 & 2x^2 - x - 1 \\ \hline
x^2 - 5 & 2x^4 - x^3 - 11x^2 + 5x + 5 \\
 & 2x^4 \qquad - 10x^2 \\
 & - \qquad\quad + \\ \hline
 & -x^3 - x^2 + 5x + 5 \\
 & -x^3 \qquad + 5x \\
 & + \qquad\quad - \\ \hline
 & -x^2 + 5 \\
 & -x^2 + 5 \\
 & + \qquad - \\ \hline
 & 0
\end{array}$$

$\therefore$ Quotient $= 2x^2 - x - 1$

$= (2x + 1)(x - 1)$

Hence, other two zeros are $-\frac{1}{2}$ and 1.

OR

Let $p(x)$ be added to $f(x) = 2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial is divisible by $(x^2 - 4x + 8)$.

Then, $f(x) + p(x) = (x^2 - 4x + 8) \times \text{quotient}$

$\Rightarrow$ $f(x) = (x^2 - 4x + 8) \times \text{quotient} - p(x)$

This shows that $p(x)$ is the negative of the remainder.

Now,

$$\begin{array}{r|l}
 & 2x + 5 \\ \hline
x^2 - 4x + 8 & 2x^3 - 3x^2 + 6x + 7 \\
 & 2x^3 - 8x^2 + 16x \\
 & - \quad + \quad - \\ \hline
 & 5x^2 - 10x + 7 \\
 & 5x^2 - 20x + 40 \\
 & + \quad + \quad - \\ \hline
 & 10x - 33
\end{array}$$

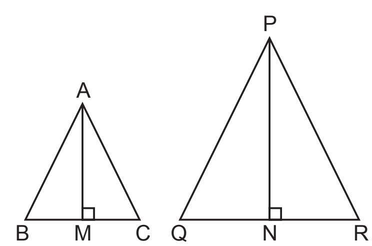
Hence, $(33 - 10x)$ is to be added.

36. We are given two Δs ABC and PQR such that

$\Delta ABC \sim \Delta PQR$

We need to prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{PQ}\right)^2 = \left(\frac{CA}{RP}\right)^2$$



Draw AM $\perp$ BC and PN $\perp$ QR.

Now, $ar(\Delta ABC) = \frac{1}{2} \times BC \times AM$; $ar(\Delta PQR) = \frac{1}{2} \times QR \times PN$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AM}{QR \times PN} \qquad \text{...(1)}$$

In Δs ABM and PQN, we have

$$\angle B = \angle Q \text{ and } \angle M = \angle N$$

So, by AA Similarity Criterion, $\Delta ABM \sim \Delta PQN$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AM}{PN} \qquad \text{...(2)}$$

Also, $\Delta ABC \sim \Delta PQR$ gives

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \qquad ...(3)$$

From (1), (2) and (3), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC}{QR} \times \frac{AM}{PN} \qquad \text{[From (1)]}$$

$$= \frac{AB}{PQ} \times \frac{AM}{PN} \qquad \text{[From (3)]}$$

$$= \frac{AB}{PQ} \times \frac{AB}{PQ} \qquad \text{[From (2)]}$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Now, using (3), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2.$$

37. Let the sides of two squares be 'x' metres and 'y' metres.

Then, $\quad x^2 + y^2 = 544 \quad$ and $\quad 4x - 4y = 32$ or $x - y = 8$

$$\Rightarrow \quad x^2 + (x-8)^2 = 544 \qquad [\because y = x - 8]$$

$$\Rightarrow \quad 2x^2 - 16x - 480 = 0$$

$$\Rightarrow \quad x^2 - 8x - 240 = 0$$

$$\Rightarrow \quad x^2 - 20x + 12x - 240 = 0$$

$$\Rightarrow \quad (x-20)(x+12) = 0$$

$$\Rightarrow \quad x - 20 = 0 \qquad [\because x + 12 \neq 0.]$$

$$\Rightarrow \quad x = 20$$

Thus, the sides of two square are 20 m and 12 m.

OR

Let the speed of the stream be 'x' km/h.

As the speed of the motorboat in still water is 18 km/h, downstream speed of the motorboat is $(x + 18)$ km/h and upstream speed of the motorboat is $(18 - x)$ km/h.

Time taken to go upstream $= \dfrac{24}{18-x}$ hours

and time taken to go downstream $= \dfrac{24}{18+x}$ hours

As per the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow \quad 24\left[\frac{18+x-18+x}{(18-x)(18+x)}\right] = 1$$

$$\Rightarrow \quad 324 - x^2 = 48x$$

$$\text{or} \quad x^2 + 48x - 324 = 0$$

$$\text{or} \quad x^2 + 54x - 6x - 324 = 0$$

$$\text{or} \quad (x+54)(x-6) = 0$$

$$\text{or} \quad x - 6 = 0 \qquad [\because x + 54 \neq 0 \text{ as } x \text{ is positive.}]$$

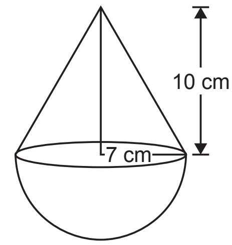
$$\text{or} \quad x = 6$$

Thus, the speed of the stream is 6 km/h.

38. Volume of the toy = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi(7)^2(10) + \frac{2}{3}\pi(7)^3$$

$$= \left(513\frac{1}{3} + 718\frac{2}{3}\right) \text{ cu cm} = 1232 \text{ cu cm}$$

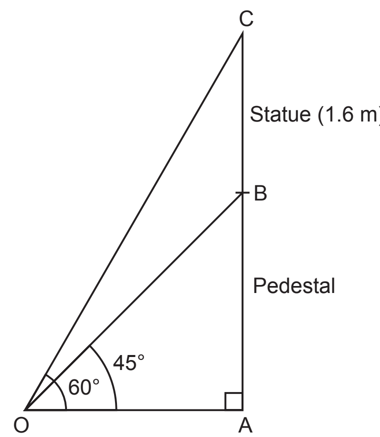


Area of the coloured sheet required = Curved surface area of the hemisphere + Curved surface area of the cone

$$= \left[2\pi(7)^2 + \pi(7)\sqrt{10^2 + 7^2}\right] \text{ sq cm}$$

$$= (308 + 268.4) \text{ sq cm} = 576.4 \text{ sq cm.}$$

39. Let AB represent the pedestal and BC, the statue.



In right ΔOAB,

$$\frac{AB}{OA} = \tan 45° = 1$$

$\Rightarrow$ $OA = AB$ ...(1)

In right ΔOAC,

$$\frac{AC}{OA} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ $OA = \dfrac{AC}{\sqrt{3}} = \dfrac{AB + 1.6}{\sqrt{3}}$ ...(2)

From (1) and (2), we have

$$AB = \frac{AB + 1.6}{\sqrt{3}}$$

$\Rightarrow$ $AB(\sqrt{3} - 1) = 1.6$

or $AB = \dfrac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{1.6 \times 2.73}{2} = \dfrac{4.368}{2} = 2.184$ m

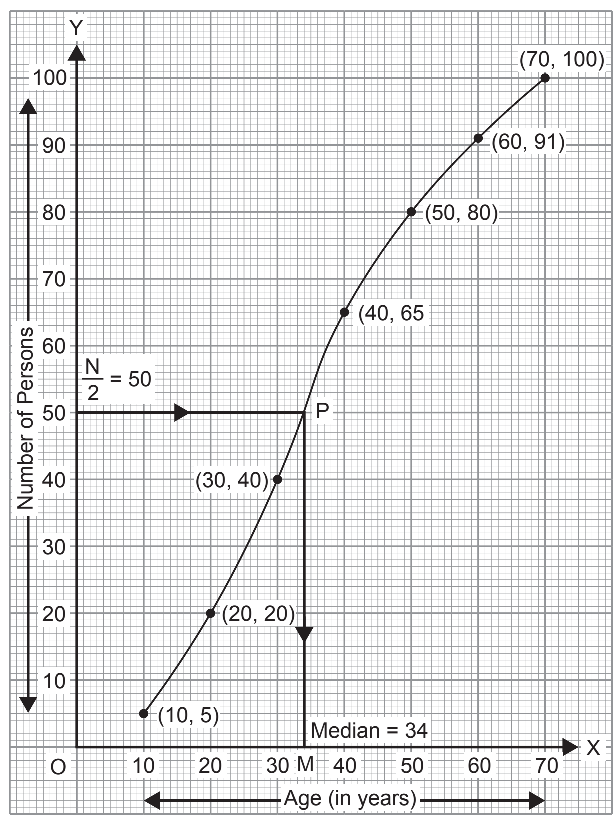
Thus, the height of the pedestal is 2.184 metres.

40. 'Less than' type cumulative frequency distribution is

Age (in years)	*No. of Persons*
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	65
Less than 50	80
Less than 60	91
Less than 70	100

The required 'ogive' is drawn in the adjoining graph.

Clearly, median of the given data is 34.



OR

Calculation of Mean:

Number of Wickets (Class)	*Class Mark* (x_i)	*Frequency* (f_i)	$u_i = \frac{x_i - A}{h}$ *where* $A = 120$	*Product* ($f_i u_i$)
20–60	40	7	–2	–14
60–100	80	5	–1	–5
100–140	120	16	0	0
140–180	160	12	1	12
180–220	200	2	2	4
220–260	240	3	3	9
Total		$\Sigma f_i = 45$		$\Sigma f_i u_i = 6$

So,
$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$
$$= 120 + \frac{6}{45} \times 40$$
$$= 125\frac{1}{3}.$$

Calculation of Median:

Classes	*Frequency*	*Cumulative Frequency*
20–60	7	7
60–100	5	12
100–140	16	28 ← Median Class
140–180	12	40
180–220	2	42
220–260	3	45

Here, $N = 45$, *i.e.*, $\frac{N}{2} = 22.5$

So, median class is 100–140.

For this class, $l = 100$, $h = 40$, $c = 12$, $\frac{N}{2} = 22.5$ and $f = 16$.

So,
$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$
$$= 100 + \frac{22.5 - 12}{16} \times 40$$
$$= 100 + 26.25$$
$$= 126.25.$$