Single Correct Choice Type

Each question in this section is a multiple choice question with four choices (A), (B), (C) and (D) out of which only one is correct.

1. For any positive integer n, define $f_n : (0, \infty) \to \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left[\frac{1}{1+(x+j)(x+j-1)}\right] \quad \text{for all } x \in (0, \infty)$$

$\Big($Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.$\Big)$

Then, which of the following statement(s) is(are) TRUE?

(A) $\sum\limits_{j=1}^{5} \tan^2\left(f_j(0)\right) = 55.$

(B) $\sum\limits_{j=1}^{10} \left(1 + f_j'(0)\right) \sec^2\left(f_j(0)\right) = 10.$

(C) For any fixed positive integer n, $\lim\limits_{x \to \infty} \tan\left(f_n(x)\right) = \frac{1}{n}$.

(D) For any fixed positive integer n, $\lim\limits_{x \to \infty} \sec^2\left(f_n(x)\right) = 1$.

[JEE 2018]

2. The equation of the plane passing through $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(A) $14x + 2y - 15z = 1$.

(B) $14x - 2y + 15z = 27$.

(C) $14x + 2y + 15z = 31$.

(D) $-14x + 2y + 15z = 3$.

[JEE 2017]

3. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$

Then the triangle PQR has S as its

(A) centroid.

(B) circumcentre.

(C) incentre.

(D) orthocenter.

[JEE 2017]

4. If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right) dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, \quad x > 0$$

and $y(0) = \sqrt{7}$, then $y(256) =$

(A) 3.

(B) 9.

(C) 16.

(D) 80.

[JEE 2017]

5. If $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

(A) $f'(1) \leq 0$. (B) $0 < f'(1) \leq \frac{1}{2}$. (C) $\frac{1}{2} < f'(1) \leq 1$. (D) $f'(1) > 1$.

[JEE 2017]

6. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(A) 126. (B) 198. (C) 162. (D) 135.

[JEE 2017]

7. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x+y+z = 10$. Then the probability that z is even, is

(A) $\frac{36}{55}$. (B) $\frac{6}{11}$. (C) $\frac{1}{2}$. (D) $\frac{5}{11}$.

[JEE 2017]

8. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

$$P(\text{computer turns out to be defective given that it is produced in plant } T_1)$$
$$= 10\, P(\text{computer turns out to be defective given that it is produced in plant } T_2),$$

where $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$. (B) $\frac{47}{79}$. (C) $\frac{78}{93}$. (D) $\frac{75}{83}$.

[JEE 2016]

9. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

(A) $\frac{1}{64}$. (B) $\frac{1}{32}$. (C) $\frac{1}{27}$. (D) $\frac{1}{25}$.

[JEE 2016]

10. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals

(A) 52. (B) 103. (C) 201. (D) 205.

[JEE 2016]

11. The value of $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x}\, dx$ is equal to

(A) $\dfrac{\pi^2}{4} - 2.$ (B) $\dfrac{\pi^2}{4} + 2.$ (C) $\pi^2 - e^{\pi/2}.$ (D) $\pi^2 + e^{\pi/2}.$

[JEE 2016]

12. Area of the region $\left\{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to

(A) $\dfrac{1}{6}.$ (B) $\dfrac{4}{3}.$ (C) $\dfrac{3}{2}.$ (D) $\dfrac{5}{3}.$

[JEE 2016]

13. Let P be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{1}$ is

(A) $x + y - 3z = 0.$ (B) $3x + z = 0.$ (C) $x - 4y + 7z = 0.$ (D) $2x - y = 0.$

[JEE 2016]

14. Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \dfrac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(A) $\displaystyle\lim_{x \to 0+} f'\left(\frac{1}{x}\right) = 1.$ (B) $\displaystyle\lim_{x \to 0+} x\, f\left(\frac{1}{x}\right) = 2.$

(C) $\displaystyle\lim_{x \to 0+} x^2\, f'(x) = 0.$ (D) $|f(x)| \leq 2$ for all $x \in (0, 2).$

[JEE 2016]

15. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}.$ (B) $1.$ (C) $-1.$ (D) $-\sqrt{2}.$

[JEE 2014]

16. The function $y = f(x)$ is the solution of the differential equation

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$$

in $(-1, 1)$ satisfying $f(0) = 0$. Then $\displaystyle\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)\, dx$ is

(A) $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}.$ (B) $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}.$

(C) $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}.$ (D) $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{2}.$

[JEE 2014]

17. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\, dx$ is equal to

(A) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}\, du.$

(B) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17}\, du.$

(C) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17}\, du.$

(D) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}\, du.$

[JEE 2014]

18. Let $f : [0,2] \to \mathbb{R}$ be a function which is continuous on $[0,2]$ and is differentiable on $(0,2)$ with $f(0) = 1$. Let

$$F(x) = \int_0^{x^2} f(\sqrt{t})\, dt$$

for $x \in [0,2]$. If $F'(x) = f'(x)$ for all $x \in (0,2)$, then $F(2)$ equals

(A) $e^2 - 1$. (B) $e^4 - 1$.

(C) $e - 1$. (D) e^4.

[JEE 2014]

19. Six cards and six envelopes are numbered $1, 2, 3, 4, 5, 6$ and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264. (B) 265.

(C) 53. (D) 67.

[JEE 2014]

20. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

(A) $\frac{1}{2}$. (B) $\frac{1}{3}$. (C) $\frac{2}{3}$. (D) $\frac{3}{4}$.

[JEE 2014]

21. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

(A) $\frac{235}{256}$. (B) $\frac{21}{256}$. (C) $\frac{3}{256}$. (D) $\frac{253}{256}$.

[JEE 2013]

22. Let $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2\, f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of

$$\int_{1/2}^{1} f(x)\, dx$$

lies in the interval

(A) $(2e - 1, 2e)$. (B) $(e - 1, 2e - 1)$. (C) $\left(\frac{e-1}{2}, e - 1\right)$. (D) $\left(0, \frac{e-1}{2}\right)$.

[JEE 2013]

23. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $4(\sqrt{2} - 1)$. (B) $2\sqrt{2}(\sqrt{2} - 1)$. (C) $2(\sqrt{2} + 1)$. (D) $2\sqrt{2}(\sqrt{2} + 1)$.

[JEE 2013]

24. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Then the equation of the curve is

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$. (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$.

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$. (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$.

[JEE 2013]

25. The value of $\cot\left[\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right]$ is

(A) $\frac{23}{25}$. (B) $\frac{25}{23}$. (C) $\frac{23}{24}$. (D) $\frac{24}{23}$.

[JEE 2013]

26. Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line

(A) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$. (B) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$.

(C) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$. (D) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$.

[JEE 2013]

27. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PR}$, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

(A) 5. (B) 20. (C) 10. (D) 30.

[JEE 2013]

28. Let $f(x) = \begin{cases} x^2 \left|\cos \dfrac{\pi}{x}\right| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

(A) differentiable both at $x = 0$ and at $x = 2$.

(B) differentiable at $x = 0$ but not differentiable at $x = 2$.

(C) not differentiable at $x = 0$ but differentiable at $x = 2$.

(D) differentiable neither at $x = 0$ nor at $x = 2$.

[JEE 2012]

29. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto.

(B) onto but not one-one.

(C) one-one but not onto.

(D) neither one-one nor onto.

[JEE 2012]

30. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10}.

(B) 2^{11}.

(C) 2^{12}.

(D) 2^{13}.

[JEE 2012]

31. The integral $\displaystyle\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}\, dx$ equals (for some arbitrary constant K)

(A) $-\dfrac{1}{(\sec x + \tan x)^{11/2}} \left[\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right] + K.$

(B) $\dfrac{1}{(\sec x + \tan x)^{11/2}} \left[\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right] + K.$

(C) $-\dfrac{1}{(\sec x + \tan x)^{11/2}} \left[\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right] + K.$

(D) $\dfrac{1}{(\sec x + \tan x)^{11/2}} \left[\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right] + K.$

[JEE 2012]

32. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is

(A) $\dfrac{1}{\sqrt{2}}$.

(B) $\sqrt{2}$.

(C) 2.

(D) $2\sqrt{2}$.

[JEE 2012]

33. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\dfrac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is

(A) $5x - 11y + z = 17$.

(B) $\sqrt{2}\, x + y = 3\sqrt{2} - 1$.

(C) $x + y + z = \sqrt{3}$.

(D) $x - \sqrt{2}\, y = 1 - \sqrt{2}$.

[JEE 2012]

34. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a}+\vec{b}) \cdot (-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0. (B) 3. (C) 4. (D) 8.

[JEE 2012]

35. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. (B) $PX = X$. (C) $PX = 2X$. (D) $PX = -X$.

[JEE 2012]

36. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$. (B) $\frac{108}{216}$.

(C) $\frac{125}{216}$. (D) $\frac{127}{216}$.

[JEE 2012]

37. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \left(x^2 + \ln \frac{\pi + x}{\pi - x} \right) dx$$

is

(A) 0. (B) $\frac{\pi^2}{2} - 4$.

(C) $\frac{\pi^2}{2} + 4$. (D) $\frac{\pi^2}{2}$.

[JEE 2012]

38. The value of

$$\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin (\ln 6 - x^2)} \, dx$$

is

(A) $\frac{1}{4} \ln \frac{3}{2}$. (B) $\frac{1}{2} \ln \frac{3}{2}$.

(C) $\ln \frac{3}{2}$. (D) $\frac{1}{6} \ln \frac{3}{2}$.

[JEE 2011]

39. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{a}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$. (B) $-3\hat{i} - 3\hat{j} - \hat{k}$.

(C) $3\hat{i} - \hat{j} + 3\hat{k}$. (D) $\hat{i} + 3\hat{j} - 3\hat{k}$.

[JEE 2011]

40. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$$

where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2. (B) 6.

(C) 4. (D) 8.

[JEE 2011]

41. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} f(x)\, dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$. (B) $R_1 = 3R_2$. (C) $2R_1 = R_2$. (D) $3R_1 = R_2$.

[JEE 2011]

42. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying

$$(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x),$$

where $(f \circ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$. (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, \ldots\}$.

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. (D) $2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

[JEE 2011]

43. Let $f : (0, 1) \to \mathbb{R}$ be defined by

$$f(x) = \frac{b - x}{1 - bx},$$

where b is a constant such that $0 < b < 1$. Then

(A) f is not invertible on $(0, 1)$. (B) $f \neq f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$.

(C) $f = f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$. (D) f^{-1} is differentiable on $(0, 1)$.

[JEE 2011]

44. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) $x + 2y - 2z = 0.$ (B) $3x + 2y - 2z = 0.$

(C) $x - 2y + z = 0.$ (D) $5x + 2y - 4z = 0.$

[JEE 2010]

45. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{\imath} - \hat{\jmath}$, $4\hat{\imath}$, $3\hat{\imath} + 3\hat{\jmath}$ and $-3\hat{\imath} + 2\hat{\jmath}$ respectively. The quadrilateral $PQRS$ must be a

(A) parallelogram, which is neither a rhombus nor a rectangle.

(B) square.

(C) rectangle, but not a square.

(D) rhombus, but not a square.

[JEE 2010]

46. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has exactly two distinct solutions, is

(A) $0.$ (B) $2^9 - 1.$ (C) $168.$ (D) $2.$

[JEE 2010]

47. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4 + 4} \, dt$ is

(A) $0.$ (B) $\frac{1}{12}.$ (C) $\frac{1}{24}.$ (D) $\frac{1}{64}.$

[JEE 2010]

48. Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by

$$f(x) = e^{x^2} + e^{-x^2}, \quad g(x) = xe^{x^2} + e^{-x^2} \quad \text{and} \quad h(x) = x^2 e^{x^2} + e^{-x^2}.$$

If a, b and c denote, respectively, the absolute maximum of f, g and h on $[0, 1]$, then

(A) $a = b$ and $c \neq b$. (B) $a = c$ and $a \neq b$. (C) $a \neq b$ and $c \neq b$. (D) $a = b = c$.

[JEE 2010]

49. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} \, dx$ is(are)

(A) $\frac{22}{7} - \pi.$ (B) $\frac{2}{105}.$ (C) $0.$ (D) $\frac{71}{15} - \frac{3\pi}{2}.$

[JEE 2010]

50. Let f be a real-valued function defined on the interval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$, for all $x \in (-1, 1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})(2)$ is equal to

(A) 1. (B) $\frac{1}{3}$. (C) $\frac{1}{2}$. (D) $\frac{1}{e}$.

[JEE 2010]

51. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$. (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$.

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$. (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$.

[JEE 2010]

52. Two adjacent sides of a parallelogram $ABCD$ are given by

$$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k} \text{ and } \overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}.$$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$. (B) $\frac{\sqrt{17}}{9}$. (C) $\frac{1}{9}$. (D) $\frac{4\sqrt{5}}{9}$.

[JEE 2010]

53. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$, respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$. (B) $\frac{6}{7}$.

(C) $\frac{20}{23}$. (D) $\frac{9}{20}$.

[JEE 2010]

54. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is

(A) $\frac{1}{4}$. (B) $-\frac{1}{4}$.

(C) $\frac{1}{8}$. (D) $-\frac{1}{8}$.

[JEE 2009]

55. Let f be a non-negative function defined on the interval $[0,1]$. If

$$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt, \quad 0 \le x \le 1,$$

and $f(0)=0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$.

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$.

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$.

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$.

[JEE 2009]

56. If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 1$ and $\vec{a}\cdot\vec{c} = \frac{1}{2}$, then

(A) $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar.

(B) $\vec{b}$, $\vec{c}$, $\vec{d}$ are non-coplanar.

(C) $\vec{b}$, $\vec{d}$ are non-parallel.

(D) $\vec{a}$, $\vec{d}$ are parallel and $\vec{b}$, $\vec{c}$ are parallel.

[JEE 2009]

57. A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals

(A) 1.

(B) $\sqrt{2}$.

(C) $\sqrt{3}$.

(D) 2.

[JEE 2009]

58. A particle P starts from the point $z_0 = 1+2i$, where $i=\sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) $6+7i$.

(B) $-7+6i$.

(C) $7+6i$.

(D) $-6+7i$.

[JEE 2008]

59. Let the function $g:(-\infty,\infty)\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is

(A) even and is strictly increasing in $(0,\infty)$.

(B) odd and is strictly decreasing in $(-\infty,\infty)$.

(C) odd and is strictly increasing in $(-\infty,\infty)$.

(D) neither even nor odd, but is strictly increasing in $(-\infty,\infty)$.

[JEE 2008]

60. The area of the region between the curves $y = \sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y = \sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \dfrac{\pi}{4}$ is

(A) $\displaystyle\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\, dt.$

(B) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\, dt.$

(C) $\displaystyle\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\, dt.$

(D) $\displaystyle\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\, dt.$

[JEE 2008]

61. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8.

(B) 3, 6 or 9.

(C) 4 or 8.

(D) 5 or 10.

[JEE 2008]

62. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$.

(B) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$.

(C) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$.

(D) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$.

[JEE 2008]

63. Let

$$I = \int \frac{e^x}{e^{4x}+e^{2x}+1}\, dx, \quad J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}\, dx.$$

Then, for an arbitrary constant C , the value of $J - I$ equals

(A) $\dfrac{1}{2}\log\left[\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right] + C.$

(B) $\dfrac{1}{2}\log\left[\dfrac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right] + C.$

(C) $\dfrac{1}{2}\log\left[\dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right] + C.$

(D) $\dfrac{1}{2}\log\left[\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right] + C.$

[JEE 2008]

64. Let $g(x) = \log f(x)$, where $f(x)$ is a twice differentiable positive function defined on $(0, \infty)$ such that $f(x+1) = x\, f(x)$. Then, for $N = 1, 2, 3, \ldots,$

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

(A) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$.

(B) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$.

(C) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N+1)^2}\right\}$.

(D) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N+1)^2}\right\}$.

[JEE 2008]

65. Consider the curves

$$C_1 : y^2 = 4x$$

$$C_2 : x^2 + y^2 - 6x + 1 = 0$$

Then,

(A) C_1 and C_2 touch each other only at one point.

(B) C_1 and C_2 touch each other exactly at two points.

(C) C_1 and C_2 intersect (but do not touch) at exactly two points.

(D) C_1 and C_2 neither intersect nor touch each other.

[JEE 2008]

66. If $0 < x < 1$, then

$$\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$$

(A) $\frac{x}{\sqrt{1+x^2}}$. (B) x. (C) $x\sqrt{1+x^2}$. (D) $\sqrt{1+x^2}$.

[JEE 2008]

67. The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}$, $\hat{b}$, $\hat{c}$ such that

$$\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = \frac{1}{2}$$

Then, the volume of the parallelepiped is

(A) $\frac{1}{\sqrt{2}}$. (B) $\frac{1}{2\sqrt{2}}$. (C) $\frac{\sqrt{3}}{2}$. (D) $\frac{1}{\sqrt{3}}$.

[JEE 2008]

68. Let $g(x) = \dfrac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x\to 1+} g(x) = p$, then

(A) $n = 1, m = 1$. (B) $n = 1, m = -1$.

(C) $n = 2, m = 2$. (D) $n > 2, m = n$.

[JEE 2008]

69. The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3 & \text{if } -3 < x \leq -1 \\ x^{2/3} & \text{if } -1 < x < 2 \end{cases}$$

is

(A) 0. (B) 1. (C) 2. (D) 3.

[JEE 2008]

70. Let f and g be real-valued functions defined on the interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x)\ \sin x$.

STATEMENT-1: $\lim_{x\to 0} [g(x)\ \cot x - g(0)\ \operatorname{cosec} x] = f''(0)$.

and

STATEMENT-2: $f'(0) = g(0)$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

[JEE 2008]

71. Consider three planes

$$P_1 : x - y + z = 1$$
$$P_2 : x + y - z = -1$$
$$P_3 : x - 3y + 3z = 2$$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1 and P_1 and P_2 respectively.

STATEMENT-1: At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT-2: The three planes do not have a common point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

[JEE 2008]

72. Consider the system of equations

$$\begin{aligned} x - 2y + 3z &= -1 \\ -x + y - 2z &= k \\ x - 3y + 4z &= 1 \end{aligned}$$

STATEMENT-1: The system of equations has no solution for $k \neq 3$.

and

STATEMENT-2: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2008]

73. Consider the system of equations

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned}$$

where a, b, c, $d \in \{0, 1\}$.

STATEMENT-1: The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT-2: The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2008]

74. Let a solution $y = y(x)$ of the differential equation

$$x\sqrt{x^2 - 1}\, dy - y\sqrt{y^2 - 1}\, dx = 0$$

satisfy $y(2) = \frac{2}{\sqrt{3}}$.

STATEMENT-1: $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$.

and

STATEMENT-2: $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2008]

75. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

for each $x > 0$. Then $f(x)$ is

(A) $\dfrac{1}{3x} + \dfrac{2x^2}{3}$. (B) $-\dfrac{1}{3x} + \dfrac{4x^2}{3}$. (C) $-\dfrac{1}{x} + \dfrac{2}{x^2}$. (D) $\dfrac{1}{x}$.

[JEE 2007]

76. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

(A) $\dfrac{1}{2}$. (B) $\dfrac{1}{3}$. (C) $\dfrac{2}{5}$. (D) $\dfrac{1}{5}$.

[JEE 2007]

77. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

(A) on the left of $x = c$. (B) on the right of $x = c$.
(C) at no point. (D) at all points.

[JEE 2007]

78. $\displaystyle\lim_{x \to \frac{\pi}{4}} \left[\frac{\displaystyle\int_2^{\sec^2 x} f(t)\, dt}{x^2 - \dfrac{\pi^2}{16}} \right]$ equals

(A) $\dfrac{8}{\pi} f(2)$. (B) $\dfrac{2}{\pi} f(2)$. (C) $\dfrac{2}{\pi} f\left(\dfrac{1}{2}\right)$. (D) $4f(2)$.

[JEE 2007]

79. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero (B) one (C) two (D) three

[JEE 2007]

80. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$. (B) $(3-4i)e^{i\pi/4}$. (C) $(4+3i)e^{i\pi/4}$. (D) $(3+4i)e^{i\pi/4}$.

[JEE 2007]

81. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals

(A) $P(E^c) + P(F^c)$. (B) $P(E^c) - P(F^c)$.

(C) $P(E^c) - P(F)$. (D) $P(E) - P(F^c)$.

[JEE 2007]

82. $\dfrac{d^2x}{dy^2}$ equals

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$. (B) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$.

(C) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$. (D) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$.

[JEE 2007]

83. The differential equation $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at $(0, 1)$.

(B) variable radii and a fixed centre at $(0, -1)$.

(C) fixed radius 1 and variable centres along the x-axis.

(D) fixed radius 1 and variable centres along the y-axis.

[JEE 2007]

84. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct?

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$.

(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$.

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$.

(D) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are mutually perpendicular.

[JEE 2007]

85. Let

$$f(x) = \frac{x}{(1+x^n)^{1/n}}$$

for $n \geq 2$ and

$$g(x) = \underbrace{(f \circ f \circ \cdots \circ f)}_{n \text{ times}}(x).$$

Then $\int x^{n-2}g(x)\,dx$ equals

(A) $\dfrac{(1+nx^n)^{1-\frac{1}{n}}}{n(n-1)}+K.$

(B) $\dfrac{(1+nx^n)^{1-\frac{1}{n}}}{n-1}+K.$

(C) $\dfrac{(1+nx^n)^{1+\frac{1}{n}}}{n(n+1)}+K.$

(D) $\dfrac{(1+nx^n)^{1+\frac{1}{n}}}{n+1}+K.$

[JEE 2007]

86. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.
STATEMENT-1: $P(H_i|E) > P(E|H_i) \cdot P(H_i)$ for $i = 1, 2, \ldots, n$.
because
STATEMENT-2: $\sum\limits_{i=1}^{n} P(H_i) = 1$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

87. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.
STATEMENT-1: $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \vec{0}$.
because
STATEMENT-2: $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

88. Let $F(x)$ be an indefinite integral of $\sin^2 x$.
STATEMENT-1: The function $F(x)$ satisfies $F(x+\pi) = F(x)$ for all real x.
because
STATEMENT-2: $\sin^2(x+\pi) = \sin^2 x$ for all real x.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

89. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
STATEMENT-1: The parametric equations of the line of intersection of the given planes are

$$x = 3 + 14t, \quad y = 1 + 2t, \quad z = 15t.$$

because
STATEMENT-2: The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

90. Let $f(x) = 2 + \cos x$ for all real x.
STATEMENT-1: For each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.
because
STATEMENT-2: $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

91. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
STATEMENT-1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.
because
STATEMENT-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

[JEE 2007]

Answers

1. (D)	**2.** (C)	**3.** (D)	**4.** (A)	**5.** (D)	**6.** (B)
7. (B)	**8.** (C)	**9.** (C)	**10.** (B)	**11.** (A)	**12.** (C)
13. (C)	**14.** (A)	**15.** (C)	**16.** (B)	**17.** (A)	**18.** (B)
19. (C)	**20.** (A)	**21.** (A)	**22.** (D)	**23.** (B)	**24.** (A)
25. (B)	**26.** (D)	**27.** (C)	**28.** (B)	**29.** (B)	**30.** (D)
31. (C)	**32.** (A)	**33.** (A)	**34.** (C)	**35.** (D)	**36.** (A)
37. (B)	**38.** (A)	**39.** (C)	**40.** (A)	**41.** (C)	**42.** (A)
43. (A)	**44.** (C)	**45.** (A)	**46.** (A)	**47.** (B)	**48.** (D)
49. (A)	**50.** (B)	**51.** (A)	**52.** (B)	**53.** (C)	**54.** (A)
55. (C)	**56.** (C)	**57.** (C)	**58.** (D)	**59.** (C)	**60.** (B)
61. (D)	**62.** (A)	**63.** (C)	**64.** (A)	**65.** (B)	**66.** (C)
67. (A)	**68.** (C)	**69.** (C)	**70.** (A)	**71.** (D)	**72.** (A)
73. (B)	**74.** (C)	**75.** (A)	**76.** (C)	**77.** (A)	**78.** (A)
79. (C)	**80.** (D)	**81.** (C)	**82.** (D)	**83.** (C)	**84.** (B)
85. (A)	**86.** (D)	**87.** (C)	**88.** (D)	**89.** (D)	**90.** (B)
91. (C)					

Multiple Correct Choices Type

Each question in this section is a multiple choice question with four choices (A), (B), (C) and (D), out of which more than one choices are correct.

1. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is(are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios $1, 2, -1$.

(B) The line
$$\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$$
is perpendicular to the line of intersection of P_1 and P_2.

(C) The acute angle between P_1 and P_2 is $60°$.

(D) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$.

[JEE 2018]

2. For every twice integrable function $f : \mathbb{R} \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is(are) TRUE?

(A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s).

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$.

(C) $\lim_{x\to\infty} f(x) = 1$.

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$.

[JEE 2018]

3. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If
$$f'(x) = \left[e^{(f(x)-g(x))}\right] g'(x) \text{ for all } x \in \mathbb{R}$$
and $f(1) = g(2) = 1$, then which of the following statement(s) is(are) TRUE?

(A) $f(2) < 1 - \log_e 2$.

(B) $f(2) > 1 - \log_e 2$.

(C) $g(1) > 1 - \log_e 2$.

(D) $g(1) < 1 - \log_e 2$.

[JEE 2018]

4. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function such that
$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\, dt$$
for all $x \in [0, \infty)$. Then, which of the following statement(s) is(are) TRUE?

(A) The curve $y = f(x)$ passes through the point $(1, 2)$.

(B) The curve $y = f(x)$ passes through the point $(2, -1)$.

(C) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1 - x^2}\}$ is $\frac{\pi - 2}{4}$.

(D) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1 - x^2}\}$ is $\frac{\pi - 1}{4}$.

[JEE 2018]

5. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1, \quad 2x - 4y + 3z = b_2, \quad x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$.

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$.

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$.

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$.

[JEE 2018]

6. Let $f : (0, \pi) \to \mathbb{R}$ be twice differentiable function such that

$$\lim_{t \to x} \frac{f(x)\ \sin t - f(t)\ \sin x}{t - x} = \sin^2 x \ \text{ for all } x \in (0, \pi).$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is(are) TRUE?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$.

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$.

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$.

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$.

[JEE 2018]

7. Let $f : \mathbb{R} \to (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval $(0, 1)$?

(A) $x^9 - f(x)$

(B) $x - \int_0^{(\pi/2)-x} f(t) \cos t \, dt$

(C) $e^x - \int_0^x f(t) \sin t \, dt$

(D) $f(x) + \int_0^{\pi/2} f(t) \sin t \, dt$

[JEE 2017]

8. Which of the following is(are) NOT the square of a 3×3 matrix with real entries?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

[JEE 2017]

9. Let x and y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

(A) $P(Y) = \frac{4}{15}$.

(B) $P(\overline{X}|Y) = \frac{1}{3}$.

(C) $P(X \cap Y) = \frac{1}{5}$.

(D) $P(X \cup Y) = \frac{2}{5}$.

[JEE 2017]

10. Let $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$, then

(A) $f(x)$ is increasing in $(0, \infty)$.

(B) $f(x)$ is decreasing in $(0, \infty)$.

(C) $f(x) > e^{2x}$ in $(0, \infty)$.

(D) $f'(x) < e^{2x}$ in $(0, \infty)$.

[JEE 2017]

11. Let $f(x) = \dfrac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\dfrac{1}{1 - x}\right)$ for $x \neq 1$. Then

(A) $\lim\limits_{x \to 1-} f(x) = 0$.

(B) $\lim\limits_{x \to 1-} f(x)$ does not exist.

(C) $\lim\limits_{x \to 1+} f(x) = 0$.

(D) $\lim\limits_{x \to 1+} f(x)$ does not exist.

[JEE 2017]

12. If $f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$.

(B) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$.

(C) $f(x)$ attains its maximum at $x = 0$.

(D) $f(x)$ attains its minimum at $x = 0$.

[JEE 2017]

13. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \leq y \leq x, 0 \leq x \leq 1\}$ into two equal parts, then

(A) $0 < \alpha \leq \frac{1}{2}$.

(B) $\frac{1}{2} < \alpha < 1$.

(C) $2\alpha^4 - 4\alpha^2 + 1 = 0$.

(D) $\alpha^4 + 4\alpha^2 - 1 = 0$.

[JEE 2017]

14. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}\, dx$, then

(A) $I > \log_e 99$. (B) $I < \log_e 99$. (C) $I < \frac{49}{50}$. (D) $I > \frac{49}{50}$.

[JEE 2017]

15. Consider a pyramid $OPQRS$ located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base $OPQR$ of the pyramid is a square with $OP = 3$. The point S is directly above the mid-point T of diagonal OQ such that $TS = 3$. Then

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$.

(B) the equation of the plane containing the triangle OQS is $x - y = 0$.

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$.

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$.

[JEE 2016]

16. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(A) $\alpha = 0, k = 8$. (B) $4\alpha - k + 8 = 0$.

(C) $\det(P \ \mathrm{adj}(Q)) = 2^9$. (D) $\det(Q \ \mathrm{adj}(P)) = 2^{13}$.

[JEE 2016]

17. A solution curve of the differential equation

$$(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0, \quad x > 0,$$

passes through the point $(1, 3)$. Then the solution curve

(A) intersects $y = x + 2$ exactly at one point. (B) intersects $y = x + 2$ exactly at two points.

(C) intersects $y = (x+2)^2$. (D) does **NOT** intersect $y = (x+3)^2$.

[JEE 2016]

18. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then

(A) $g'(2) = \frac{1}{15}$. (B) $h'(1) = 666$. (C) $h(0) = 16$. (D) $h(g(3)) = 36$.

[JEE 2016]

19. Let $f(x) = \lim_{n\to\infty}\left[\frac{n^n(x+n)\left(x+\frac{n}{2}\right)\dots\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right)\dots\left(x^2+\frac{n^2}{n^2}\right)}\right]$, for all $x>0$. Then

(A) $f\left(\frac{1}{2}\right) \geq f(1)$.

(B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$.

(C) $f'(2) \leq 0$.

(D) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$.

[JEE 2016]

20. Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b\,|x|\,\sin(|x^3 + x|)$. Then f is

(A) differentiable at $x = 0$ if $a = 0$ and $b = 1$.
(B) differentiable at $x = 1$ if $a = 1$ and $b = 0$.
(C) **NOT** differentiable at $x = 0$ if $a = 1$ and $b = 0$.
(D) **NOT** differentiable at $x = 1$ if $a = 1$ and $b = 1$.

[JEE 2016]

21. Let $f : \mathbb{R} \to (0,\infty)$ and $g : \mathbb{R} \to \mathbb{R}$ be twice differentiable functions such that f'' and g'' are continuous functions on $\mathbb{R}$. Suppose $f'(2) = g(2) = 0$, $f''(2) \neq 0$ and $g'(2) \neq 0$.
If $\lim_{x\to 2}\frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(A) f has a local minimum at $x = 2$.
(B) f has a local maximum at $x = 2$.
(C) $f''(2) > f(2)$.
(D) $f(x) - f''(x) = 0$ for at least one $x \in \mathbb{R}$.

[JEE 2016]

22. Let $f : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ be functions defined by

$$f(x) = \lfloor x^2 - 3\rfloor \text{ and } g(x) = |x|\,f(x) + |4x-7|\,f(x),$$

where $\lfloor y \rfloor$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(A) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$.

(B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$.

(C) g is **NOT** differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$.

(D) g is **NOT** differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$.

[JEE 2016]

23. Let a, λ, $\mu \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$
$$3x - 2y = \mu$$

Which of the following statement(s) is(are) correct?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ.

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ.

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$.

[JEE 2016]

24. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w} = \dfrac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u} \times \vec{v}| = 1$ and $\hat{w} \cdot (\hat{u} \times \vec{v}) = 1$. Which of the following statement(s) is(are) correct?

(A) There is exactly one choice for such $\vec{v}$.

(B) There are infinitely many choices for such $\vec{v}$.

(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$.

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$.

[JEE 2016]

25. Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew-symmetric?

(A) $Y^3Z^4 - Z^4Y^3$. (B) $X^{44} + Y^{44}$. (C) $X^4Z^3 - Z^3X^4$. (D) $X^{23} + Y^{23}$.

[JEE 2015]

26. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha?$$

(A) -4. (B) 9. (C) -9. (D) 4.

[JEE 2015]

27. In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point $(0, 1, 0)$ from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$. (B) $2\alpha - \beta + 2\gamma + 4 = 0$.

(C) $2\alpha + \beta - 2\gamma - 10 = 0$. (D) $2\alpha - \beta + 2\gamma - 8 = 0$.

[JEE 2015]

28. In $\mathbb{R}^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$. (B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$. (C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$. (D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$.

[JEE 2015]

29. Let $y(x)$ be a solution of the differential equation

$$(1 + e^x)y' + ye^x = 1.$$

If $y(0) = 2$, then which of the following statements is (are) true?

(A) $y(-4) = 0$. (B) $y(-2) = 0$.

(C) $y(x)$ has a critical point in the interval $(-1, 0)$.

(D) $y(x)$ has no critical point in the interval $(-1, 0)$.

[JEE 2015]

30. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' $\left(\text{here } y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true?

(A) $P = y + x$. (B) $P = y - x$.

(C) $P + Q = 1 - x + y + y' + (y')^2$. (D) $P - Q = x + y - y' - (y')^2$.

[JEE 2015]

31. Let $g : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let

$$f(x) = \begin{cases} \frac{x}{|x|}\, g(x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

(A) f is differentiable at $x = 0$. (B) h is differentiable at $x = 0$.

(C) $f \circ h$ is differentiable at $x = 0$. (D) $h \circ f$ is differentiable at $x = 0$.

[JEE 2015]

32. Let $f(x) = \sin\left(\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right)$ for all $x \in \mathbb{R}$ and $g(x) = \frac{\pi}{2} \sin x$ for all $x \in \mathbb{R}$. Let $(f \circ g)(x)$ denote $f(g(x))$ and $(g \circ f)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

(A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. (B) Range of $f \circ g$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

(C) $\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \dfrac{\pi}{6}$.

(D) There is an $x \in \mathbb{R}$ such that $(g \circ f)(x) = 1$.

[JEE 2015]

33. Let PQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}\cdot\vec{c} = 24$, then which of the following is (are) true?

(A) $\dfrac{|\vec{c}|^2}{2} - |\vec{a}| = 12$.

(B) $\dfrac{|\vec{c}|^2}{2} + |\vec{a}| = 30$.

(C) $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$.

(D) $\vec{a}\cdot\vec{b} = -72$.

[JEE 2015]

34. If $\alpha = 3\sin^{-1}\left(\dfrac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\dfrac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are) .

(A) $\cos\beta > 0$.

(B) $\sin\beta < 0$.

(C) $\cos(\alpha+\beta) > 0$.

(D) $\cos\alpha < 0$.

[JEE 2015]

35. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\displaystyle\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)\,dt}{\displaystyle\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\,dt} = L?$$

(A) $a = 2$, $L = \dfrac{e^{4\pi}-1}{e^{\pi}-1}$.

(B) $a = 2$, $L = \dfrac{e^{4\pi}+1}{e^{\pi}+1}$.

(C) $a = 4$, $L = \dfrac{e^{4\pi}-1}{e^{\pi}-1}$.

(D) $a = 4$, $L = \dfrac{e^{4\pi}+1}{e^{\pi}+1}$.

[JEE 2015]

36. Let $f, g : [-1, 2] \to \mathbb{R}$ be continuous functions which are twice differentiable on the interval $(-1, 2)$. Let the values of f and g at the points -1, 0 and 2 be as given in the following table:

	$x=-1$	$x=0$	$x=2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals $(-1, 0)$ and $(0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is(are)

(A) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0)\cup(0, 2)$.

(B) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$.

(C) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(0, 2)$.

(D) $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)$.

[JEE 2015]

37. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is (are)

(A) $\int_0^{\pi/4} x\, f(x)\, dx = \frac{1}{12}$.
(B) $\int_0^{\pi/4} f(x)\, dx = 0$.
(C) $\int_0^{\pi/4} x\, f(x)\, dx = \frac{1}{6}$.
(D) $\int_0^{\pi/4} f(x)\, dx = 1$.

[JEE 2015]

38. Let $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$. If $m \le \int_{1/2}^{1} f(x)\, dx \le M$, then the possible values of m and M are

(A) $m = 13, M = 24$.
(B) $m = \frac{1}{4}, M = \frac{1}{2}$.
(C) $m = -11, M = 0$.
(D) $m = 1, M = 12$.

[JEE 2015]

39. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \ne N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0.
(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix.
(C) determinant of $(M^2 + MN^2) \ge 1$.
(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix.

[JEE 2014]

40. For every pair of continuous functions $f, g : [0,1] \to \mathbb{R}$ such that

$$\max\{f(x) : x \in [0,1]\} = \max\{g(x) : x \in [0,1]\},$$

the correct statement(s) is (are):

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$.
(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$.
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$.
(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$.

[JEE 2014]

41. Let $f : (0,\infty) \to \mathbb{R}$ be given by

$$f(x) = \int_{1/x}^{x} \frac{1}{t}\, e^{-\left(t+\frac{1}{t}\right)}\, dt.$$

Then

(A) $f(x)$ is monotonically increasing on $[1,\infty)$.
(B) $f(x)$ is monotonically decreasing on $(0,1)$.
(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0,\infty)$.
(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$.

[JEE 2014]

42. Let $f:[a,b]\to[1,\infty)$ be a continuous function and let $g:\mathbb{R}\to\mathbb{R}$ be defined as

$$g(x)=\begin{cases}0 & \text{if } x<a\\ \displaystyle\int_a^x f(t)\,dt & \text{if } a\le x<b\\ \displaystyle\int_a^b f(t)\,dt & \text{if } x>b\end{cases}.$$

Then

(A) $g(x)$ is continuous but not differentiable at a.

(B) $g(x)$ is differentiable on $\mathbb{R}$.

(C) $g(x)$ is continuous but not differentiable at b.

(D) $g(x)$ is continuous and differentiable at either a or b but not both.

[JEE 2014]

43. Let $f:\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)\to\mathbb{R}$ be given by $f(x)=[\log(\sec x+\tan x)]^3$. Then

(A) $f(x)$ is an odd function.

(B) $f(x)$ is a one-one function.

(C) $f(x)$ is an onto function.

(D) $f(x)$ is an even function.

[JEE 2014]

44. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\dfrac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(A) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$.

(B) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$.

(C) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$.

(D) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$.

[JEE 2014]

45. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M.

(B) the second row of M is the transpose of the first column of M.

(C) M is a diagonal matrix with non-zero entries in the main diagonal.

(D) the product of entries in the main diagonal of M is not the square of an integer.

[JEE 2014]

46. For 3×3 matrices M and N, which of the following statement(s) is (are) NOT correct?

(A) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric.

(B) $MN-NM$ is skew symmetric for all symmetric matrices M and N.

(C) MN is symmetric for all symmetric matrices M and N.

(D) $(\operatorname{adj} M)(\operatorname{adj} N)=\operatorname{adj}(MN)$ for all invertible matrices M and N.

[JEE 2014]

47. Let $f(x) = x \sin \pi x$, $x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at

(A) a unique point in the interval $\left(n, n + \frac{1}{2}\right)$.

(B) a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$.

(C) a unique point in the interval $(n, n + 1)$.

(D) two points in the interval $(n, n + 1)$.

[JEE 2013]

48. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8 : 15$ is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(A) 24. (B) 32. (C) 45. (D) 60.

[JEE 2013]

49. A line l passing through the origin is perpendicular to the lines

$$l_1 : (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}, \quad -\infty < t < \infty$$

$$l_2 : (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, \quad -\infty < s < \infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$. (B) $(-1, -1, 0)$.

(C) $(1, 1, 1)$. (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$.

[JEE 2013]

50. The function $f(x) = 2|x| + |x + 2| - \big||x + 2| - 2|x|\big|$ has a local minimum or a local maximum at $x =$

(A) -2. (B) $-\frac{2}{3}$.

(C) 2. (D) $\frac{2}{3}$.

[JEE 2013]

51. Two lines $L_1 : x = 5, \frac{y}{3 - \alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2 - \alpha}$ are coplanar. Then α can take value(s)

(A) 1. (B) 2.

(C) 3. (D) 4.

[JEE 2013]

52. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

(A) $S \geq \frac{1}{e}$.

(B) $S \geq 1 - \frac{1}{e}$.

(C) $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$.

(D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$.

[JEE 2012]

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

(A) $P[X_1^c|X] = \frac{3}{16}$.

(B) $P[\text{Exactly two engines of the ship are functioning}|X] = \frac{7}{8}$.

(C) $P[X|X_2] = \frac{5}{16}$.

(D) $P[X|X_1] = \frac{7}{16}$.

[JEE 2012]

54. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$.

(B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$.

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$.

(D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$.

[JEE 2012]

55. For every integer n, let a_n and b_n be real numbers. Let function $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x & \text{for } x \in (2n-1, 2n) \end{cases}$$

for all integers n. If f is continuous, then which of the following hold(s) for all n?

(A) $a_{n-1} - b_{n-1} = 0$.

(B) $a_n - b_n = 1$.

(C) $a_n - b_{n+1} = 1$.

(D) $a_{n-1} - b_n = -1$.

[JEE 2012]

56. If

$$f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt, \text{ for all } x \in (0, \infty),$$

then

(A) f has a local maximum at $x = 2$.

(B) f is decreasing on $(2, 3)$.

(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$.

(D) f has a local minimum at $x = 3$.

[JEE 2012]

57. If the straight lines $\dfrac{x-1}{2} = \dfrac{y+1}{k} = \dfrac{z}{2}$ and $\dfrac{x+1}{5} = \dfrac{y+1}{2} = \dfrac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$. (B) $y + z = -1$. (C) $y - z = -1$. (D) $y - 2z = -1$.

[JEE 2012]

58. Let X and Y be two events such that $P(X|Y) = \dfrac{1}{2}$, $P(Y|X) = \dfrac{1}{3}$ and $P(X \cap Y) = \dfrac{1}{6}$. Which of the following is (are) correct?

(A) $P(X \cup Y) = \dfrac{2}{3}$. (B) X and Y are independent.

(C) X and Y are not independent. (D) $P(X^c \cap Y) = \dfrac{1}{3}$.

[JEE 2012]

59. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is(are)

(A) -2. (B) -1. (C) 1. (D) 2.

[JEE 2012]

60. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are

(A) $\hat{j} - \hat{k}$. (B) $-\hat{i} + \hat{j}$. (C) $\hat{i} - \hat{j}$. (D) $-\hat{j} + \hat{k}$.

[JEE 2011]

61. If

$$f(x) = \begin{cases} -x - \dfrac{\pi}{2} & \text{if } x \le -\dfrac{\pi}{2} \\ -\cos x & \text{if } -\dfrac{\pi}{2} < x \le 0 \\ x - 1 & \text{if } 0 < x \le 1 \\ \ln x & \text{if } x > 1 \end{cases}$$

then

(A) $f(x)$ is continuous at $x = \dfrac{\pi}{2}$. (B) $f(x)$ is not differentiable at $x = 0$.

(C) $f(x)$ is differentiable at $x = 1$. (D) $f(x)$ is differentiable at $x = -\dfrac{3}{2}$.

[JEE 2011]

62. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$.

(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$.

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$.

(D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$.

[JEE 2011]

63. Let f be a real-valued function defined on the interval $(0, \infty)$ by

$$f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}\, dt.$$

Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$.

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$.

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$.

(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$.

[JEE 2010]

64. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(A) $e - 1$.

(B) $\int_1^e \ln(e + 1 - y)\, dy$.

(C) $e - \int_0^1 e^x\, dx$.

(D) $\int_1^e \ln y\, dy$.

[JEE 2009]

65. If

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x) \sin x}\, dx, \quad n = 0, 1, 2, \ldots,$$

then

(A) $I_n = I_{n+2}$.

(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$.

(C) $\sum_{m=1}^{10} I_{2m} = 0$.

(D) $I_n = I_{n+1}$.

[JEE 2009]

66. For the function

$$f(x) = x\cos\frac{1}{x}, \quad x \geq 1,$$

(A) at least one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$.

(B) $\lim\limits_{x\to\infty} f'(x) = 1$.

(C) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$.

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$.

[JEE 2009]

67. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that

$$f(x) = f(1-x) \text{ and } f'\left(\frac{1}{4}\right) = 0.$$

Then

(A) $f''(x)$ vanishes at least twice on $[0, 1]$.

(B) $f'\left(\frac{1}{2}\right) = 0$.

(C) $\displaystyle\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\,dx = 0$.

(D) $\displaystyle\int_{0}^{1/2} f(t)\,e^{\sin \pi t}\,dt = \int_{1/2}^{1} f(1-t)\,e^{\sin\pi t}\,dt$.

[JEE 2008]

Answers

1. (C), (D)	**2.** (A), (B), (D)	**3.** (B), (C)	**4.** (B), (C)
5. (A), (D)	**6.** (B), (C), (D)	**7.** (A), (B)	**8.** (B), (D)
9. (A), (B)	**10.** (A), (C)	**11.** (A), (D)	**12.** (B), (C)
13. (B), (C)	**14.** (B), (D)	**15.** (B), (C)	**16.** (A), (B)
17. (A), (D)	**18.** (B), (C)	**19.** (B), (C), (D)	**20.** (B), (C)
21. (A), (D)	**22.** (B), (C)	**23.** (B), (C), (D)	**24.** (B), (C)
25. (C), (D)	**26.** (B), (C)	**27.** (B), (D)	**28.** (A), (B)
29. (A), (C)	**30.** (B), (C)	**31.** (A), (D)	**32.** (A), (B), (C)
33. (A), (C), (D)	**34.** (B), (C), (D)	**35.** (A), (C)	**36.** (B), (C)
37. (A), (B)	**38.** (D)	**39.** (A), (B)	**40.** (A), (D)
41. (A), (C), (D)	**42.** (A), (C)	**43.** (A), (B), (C)	**44.** (A), (B), (C)
45. (C), (D)	**46.** (C), (D)	**47.** (B), (C)	**48.** (A), (C)
49. (B), (D)	**50.** (A), (B)	**51.** (A), (D)	**52.** (A), (B), (D)
53. (B), (D)	**54.** (A), (D)	**55.** (B), (D)	**56.** (A), (B), (C), (D)
57. (A), (D)	**58.** (A), (B)	**59.** (A), (D)	**60.** (A), (D)
61. (A), (B), (C), (D)	**62.** (A), (D)	**63.** (B), (C)	**64.** (B), (C), (D)
65. (A), (B), (C)	**66.** (B), (C), (D)	**67.** (A), (B), (C), (D)	

Numerical Valued Answer Type

Each question in this section, when worked out will result in a numerical value.

1. The number of real solutions of the equation

$$\sin^{-1}\left[\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right] = \frac{\pi}{2} - \cos^{-1}\left[\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right]$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is **[JEE 2018]**

2. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b} = 0$. For some $x, y \in \mathbb{R}$, let $\vec{c} = x\,\vec{a} + y\,\vec{b} + \left(\vec{a}\times\vec{b}\right)$. If $|\vec{c}| = 2$, and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is **[JEE 2018]**

3. The value of the integral

$$\int_0^{1/2} \frac{1+\sqrt{3}}{\left[(x+1)^2(1-x)^6\right]^{1/4}}\,dx$$

is **[JEE 2018]**

4. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is **[JEE 2018]**

5. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is **[JEE 2018]**

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2+5y)(5y-2),$$

then the value of $\lim_{x\to\infty} f(x)$ is **[JEE 2018]**

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 1$. and satisfying the equation

$$f(x+y) = f(x)f'(y) + f'(x)f(y) \text{ for all } x, y \in \mathbb{R}.$$

Then, the value of $\log_e(f(4))$ is **[JEE 2018]**

8. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is **[JEE 2018]**

9. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where $O(0,0,0)$ is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is **[JEE 2018]**

10. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f(0)=0$, $f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$. If

$$g(x)=\int_x^{\pi/2}\left[f'(t)\ \operatorname{cosec} t-\cot t\ \operatorname{cosec} t\ f(t)\right]\,dt$$

for $x\in\left(0,\frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x)=$ **[JEE 2017]**

11. For a real number α, if the system

$$\begin{bmatrix}1 & \alpha & \alpha^2\\ \alpha & 1 & \alpha\\ \alpha^2 & \alpha & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1+\alpha+\alpha^2=$ **[JEE 2017]**

12. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}=$ **[JEE 2017]**

13. The total number of distinct $x\in\mathbb{R}$ for which $\begin{vmatrix}x & x^2 & 1+x^3\\ 2x & 4x^2 & 1+8x^3\\ 3x & 9x^2 & 1+27x^3\end{vmatrix}=10$ is **[JEE 2016]**

14. The total number of distinct $x\in[0,1]$ for which $\int_0^x \frac{t^2}{1+t^4}\,dt=2x-1$ is **[JEE 2016]**

15. Let $\alpha,\beta\in\mathbb{R}$ be such that $\lim\limits_{x\to 0}\left[\frac{x^2\sin(\beta x)}{\alpha x-\sin x}\right]=1$. Then $6(\alpha+\beta)$ equals **[JEE 2016]**

16. Let $z=\frac{-1+\sqrt{3}\,i}{2}$, where $i=\sqrt{-1}$, and $r,s\in\{1,2,3\}$. Let $P=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r,s) for which $P^2=-I$ is **[JEE 2016]**

17. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[JEE 2015]**

18. If the normals of the parabola $y^2=4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of r^2 is **[JEE 2015]**

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \begin{cases} \lfloor x \rfloor & \text{if } x \le 2 \\ 0 & \text{if } x > 2 \end{cases}$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x. If $I = \displaystyle\int_{-1}^{2} \frac{x\, f(x^2)}{2 + f(x+1)}\, dx$, then the value of $(4I - 1)$ is **[JEE 2015]**

20. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\dfrac{V}{250\pi}$ is **[JEE 2015]**

21. Let $F(x) = \displaystyle\int_{x}^{x^2+\pi/6} 2\cos^2 t\, dt$ for all $x \in \mathbb{R}$ and $f : \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by $x = 0$, $y = 0$, $y = f(x)$ and $x = a$, then $f(0)$ is **[JEE 2015]**

22. If $\alpha = \displaystyle\int_{0}^{1} e^{9x + 3\tan^{-1}x} \left(\frac{12 + 9x^2}{1 + x^2}\right)$, where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e |1 + \alpha| - \dfrac{3\pi}{4}\right)$ is **[JEE 2015]**

23. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(l) = \dfrac{1}{2}$. Suppose that $F(x) = \displaystyle\int_{-1}^{x} f(t)\, dt$ for all $x \in [-1, 2]$ and $G(x) = \displaystyle\int_{-1}^{x} t|f(f(t))|\, dt$ for all $x \in [-1, 2]$. If $\displaystyle\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\dfrac{1}{2}\right)$ is **[JEE 2015]**

24. Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p} + \vec{q} + \vec{r})$, $(\vec{p} - \vec{q} + \vec{r})$ and $(-\vec{p} - \vec{q} + \vec{r})$ are x, y and z, respectively, then the value of $2x + y + z$ is **[JEE 2015]**

25. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \le 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}.$$

The number of points at which $h(x)$ is not differentiable is **[JEE 2014]**

26. The value of

$$\int_{0}^{1} 4x^3 \left\{\frac{d^2}{dx^2}(1 - x^2)^5\right\} dx$$

is **[JEE 2014]**

27. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point $(1, 3)$ is **[JEE 2014]**

28. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation

$$f(x) = \frac{10 - x}{10}$$

is **[JEE 2014]**

29. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$ is **[JEE 2014]**

30. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\dfrac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\dfrac{p^2 + 2q^2 + r^2}{q^2}$ is **[JEE 2014]**

31. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$ **[JEE 2013]**

32. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that one of the events E_1, E_2 or E_3 occurs satisfy the equations

$$(\alpha - 2\beta)p = \alpha\beta \text{ and } (\beta - 3\gamma)p = 2\beta\gamma.$$

All the given probabilities are assumed to lie in the interval $(0, 1)$. Then

$$\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$$

[JEE 2013]

33. Consider the set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE 2013]**

34. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[JEE 2012]**

35. The value of $6 + \log_{3/2}\left(\dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\cdots}}}\right)$ is **[JEE 2012]**

36. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is **[JEE 2012]**

37. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is **[JEE 2012]**

38. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is **[JEE 2011]**

39. Let $f(\theta) = \sin\left[\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right]$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of

$$\frac{d}{d(\tan\theta)}(f(\theta))$$

is **[JEE 2011]**

40. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{d}{dx}f(x)$ and $g(x)$ is a given non-constant differentiable function on $\mathbb{R}$ with $g(0) = g(2) = 0$. Then the value of $y(2)$ is **[JEE 2011]**

41. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix},\ \text{and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is **[JEE 2011]**

42. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is **[JEE 2011]**

43. The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is **[JEE 2010]**

44. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of

$$\left(2\vec{a} + \vec{b}\right) \cdot \left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$$

is **[JEE 2010]**

45. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

is $\sqrt{6}$, then $|d|$ is **[JEE 2010]**

46. For any real number x, let $\lfloor x \rfloor$ denote the largest integer less than or equal to x. Let f be a real-valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - \lfloor x \rfloor & \text{if } \lfloor x \rfloor \text{ is odd} \\ 1 + \lfloor x \rfloor - x & \text{if } \lfloor x \rfloor \text{ is even} \end{cases}$$

Then the value of

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\, dx$$

is **[JEE 2010]**

47. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

is equal to **[JEE 2010]**

48. Let f be a real-valued differentiable function on $\mathbb{R}$ (the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to **[JEE 2010]**

49. Let f be a function defined on $\mathbb{R}$ (the set of all real numbers) such that

$$f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4, \text{ for all } x \in \mathbb{R}.$$

If g is a function defined on $\mathbb{R}$ with values in the interval $(0, \infty)$ such that

$$f(x) = \ln(g(x)), \text{ for all } x \in \mathbb{R},$$

then the number of points in $\mathbb{R}$ at which g has a local maximum is **[JEE 2010]**

50. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $\lfloor k \rfloor$ is equal to

[**Note:** adj M denotes the adjoint of a square matrix M and $\lfloor k \rfloor$ denotes the largest integer less than or equal to k.] **[JEE 2010]**

51. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x : x^2 + 20 \le 9x\}$ is **[JEE 2009]**

52. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0$$

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is **[JEE 2009]**

53. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30°$. The absolute value of the difference between the areas of these triangles is **[JEE 2009]**

54. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and

$$\lim_{x \to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2.$$

Then the value of $p(2)$ is **[JEE 2009]**

55. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function which satisfies

$$f(x)=\int_0^x f(t)\,dt.$$

Then the value of $f(\ln 5)$ is **[JEE 2009]**

56. If the function $f(x)=x^3+e^{x/2}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is **[JEE 2009]**

Answers

1. 2	**2.** 3	**3.** 2	**4.** 4	**5.** 119
6. 0.4	**7.** 2	**8.** 8	**9.** 0.5	**10.** 2
11. 1	**12.** 5	**13.** 2	**14.** 1	**15.** 7
16. 1	**17.** 8	**18.** 2	**19.** 0	**20.** 4
21. 3	**22.** 9	**23.** 7	**24.** 9	**25.** 3
26. 2	**27.** 8	**28.** 3	**29.** 6	**30.** 4
31. 5	**32.** 6	**33.** 5	**34.** 5	**35.** 4
36. 9	**37.** 3	**38.** 8	**39.** 1	**40.** 0
41. 9	**42.** 9	**43.** 2	**44.** 5	**45.** 6
46. 4	**47.** 1	**48.** 9	**49.** 1	**50.** 4
51. 7	**52.** 7	**53.** 4	**54.** 0	**55.** 0
56. 2				

This section contains paragraphs each describing theory, experiments, data, etc. Questions on each paragraph are given. Each question has one or more than one correct answer among the four given options (A), (B), (C) and (D).

Paragraph for Question Nos. 1 and 2

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

1. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(A) $\frac{3}{40}$. (B) $\frac{1}{8}$. (C) $\frac{7}{40}$. (D) $\frac{1}{5}$.

2. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$. (B) $\frac{1}{10}$. (C) $\frac{7}{60}$. (D) $\frac{1}{5}$.

[JEE 2018]

Paragraph for Question Nos. 3 to 5

Answer the questions by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

3. Which of the following options is the only CORRECT combination?

(A) (I) (i) (P). (B) (II) (ii) (Q).

(C) (III) (iii) (R). (D) (IV) (iv) (S).

4. Which of the following options is the only CORRECT combination?

(A) (I) (ii) (R). (B) (II) (iii) (S).

(C) (III) (iv) (P). (D) (IV) (i) (S).

5. Which of the following options is the only **INCORRECT** combination?

(A) (I) (iii) (P). (B) (II) (iv) (Q). (C) (III) (i) (R). (D) (II) (iii) (P).

[JEE 2017]

Paragraph for Question Nos. 6 and 7

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

6. $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

(A) $\sin(P+Q)$. (B) $\sin 2R$. (C) $\sin(P+R)$. (D) $\sin(Q+R)$.

7. If the triangle PQR varies, then the minimum value of

$$\cos(P+Q) + \cos(Q+R) + \cos(R+P)$$

is

(A) $-\frac{5}{3}$. (B) $-\frac{3}{2}$. (C) $\frac{3}{2}$. (D) $\frac{5}{3}$.

[JEE 2017]

Paragraph for Question Nos. 8 and 9

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games.

8. $P(X > Y)$ is

(A) $\frac{1}{4}$. (B) $\frac{5}{12}$. (C) $\frac{1}{2}$. (D) $\frac{7}{12}$.

9. $P(X = Y)$ is

(A) $\frac{11}{36}$. (B) $\frac{1}{3}$. (C) $\frac{13}{36}$. (D) $\frac{1}{2}$.

[JEE 2016]

Paragraph for Question Nos. 10 and 11

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

10. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$. (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$.

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$. (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$.

11. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$. (B) $n_1 = 2$ and $n_2 = 3$.

(C) $n_1 = 10$ and $n_2 = 20$. (D) $n_1 = 3$ and $n_2 = 6$.

[JEE 2015]

Paragraph for Question Nos. 12 and 13

Let $F : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F'(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = x\,F(x)$ for all $x \in \mathbb{R}$.

12. The correct statement(s) is(are)

(A) $f'(1) < 0$. (B) $f(2) < 0$.

(C) $f'(x) \neq 0$ for any $x \in (1, 3)$. (D) $f'(x) = 0$ for some $x \in (1, 3)$.

13. If $\displaystyle\int_1^3 x^2\,F'(x)\,dx = -12$ and $\displaystyle\int_1^3 x^3\,F''(x)\,dx = 40$, then the correct expression(s) is(are)

(A) $9f'(3) + f'(1) - 32 = 0$. (B) $\displaystyle\int_1^3 f(x)\,dx = 12$.

(C) $9f'(3) - f'(1) + 32 = 0$. (D) $\displaystyle\int_1^3 f(x)\,dx = -12$.

[JEE 2015]

Paragraph for Question Nos. 14 and 15

Given that for each $a \in (0, 1)$,

$$\lim_{h \to 0+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$$

exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$.

14. The value of $g\left(\frac{1}{2}\right)$ is

(A) π. (B) 2π.

(C) $\frac{\pi}{2}$. (D) $\frac{\pi}{4}$.

15. The value of $g'\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$. (B) π. (C) $-\frac{\pi}{2}$. (D) 0.

[JEE 2014]

Paragraph for Question Nos. 16 and 17

Box 1 contains three cards bearing numbers $1, 2, 3$; box 2 contains five cards bearing numbers $1, 2, 3, 4, 5$; and box 3 contains seven cards bearing numbers $1, 2, 3, 4, 5, 6, 7$. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.

16. The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$. (B) $\frac{53}{105}$. (C) $\frac{57}{105}$. (D) $\frac{1}{2}$.

17. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(A) $\frac{9}{105}$. (B) $\frac{10}{105}$. (C) $\frac{11}{105}$. (D) $\frac{7}{105}$.

[JEE 2014]

Paragraph for Question Nos. 18 and 19

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

18. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3 the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$. (B) $\frac{90}{648}$. (C) $\frac{558}{648}$. (D) $\frac{566}{648}$.

19. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_1 is

(A) $\frac{116}{181}$. (B) $\frac{126}{181}$. (C) $\frac{65}{181}$. (D) $\frac{55}{181}$.

[JEE 2013]

Paragraph for Question Nos. 20 and 21

Let $f : [0, 1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$.

20. Which of the following is true for $0 < x < 1$?

(A) $0 < f(x) < \infty$. (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$. (C) $-\frac{1}{4} < f(x) < 1$. (D) $-\infty < f(x) < 0$.

21. If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true?

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$. (B) $f'(x) > f(x), 0 < x < \frac{1}{4}$.

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$. (D) $f'(x) < f(x), \frac{3}{4} < x < 1$.

[JEE 2013]

Paragraph for Question Nos. 22 and 23

Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let

$$g(x) = \int_1^x \left[\frac{2(t-1)}{t+1} - \ln t\right] f(t)\, dt, \text{ for all } x \in (1, \infty).$$

22. Which of the following is true?

(A) g is increasing on $(1, \infty)$.
(B) g is decreasing on $(1, \infty)$.
(C) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$.
(D) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$.

23. Consider the statements:
P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$
Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$
Then

(A) both **P** and **Q** are true. (B) **P** is true and **Q** is false.

(C) **P** is false and **Q** is true. (D) both **P** and **Q** are false.

[JEE 2012]

Paragraph for Question Nos. 24 to 26

Let a, b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}. \qquad \text{...(E)}$$

24. If point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then value of $7a + b + c$ is

(A) 0. (B) 12. (C) 7. (D) 6.

25. Let ω be a solution of $x^3 - 1 = 0$ with $\mathrm{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) -2. (B) 2.

(C) 3. (D) -3.

26. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2+bx+c = 0$, then

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$$

is

(A) 6. (B) 7.

(C) $\frac{6}{7}$. (D) ∞.

[JEE 2011]

Paragraph for Question Nos. 27 and 28

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

27. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$. (B) $\frac{23}{30}$.

(C) $\frac{19}{30}$. (D) $\frac{11}{30}$.

28. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$. (B) $\frac{11}{23}$.

(C) $\frac{15}{23}$. (D) $\frac{12}{23}$.

[JEE 2011]

Paragraph for Question Nos. 29 to 31

Let p be an odd prime number and T_p be the following set of 2×2 matrices:

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$

29. The number of A in T_p such that A is either symmetric or skew symmetric or both. and $\det(A)$ divisible by p is

(A) $(p-1)^2$.
(B) $2(p-1)$.
(C) $(p-1)^2+1$.
(D) $2p-1$.

30. The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is
[**Note:** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$.
(B) $p^3-(p-1)^2$.
(C) $(p-1)^2$.
(D) $(p-1)(p^2-2)$.

31. The number of A in T_p such that $\det(A)$ is not divisible by p is

(A) $2p^2$.
(B) p^3-5p.
(C) p^3-3p.
(D) p^3-p^2.

[JEE 2010]

Paragraph for Question Nos. 32 to 34

Let $\mathscr{A}$ be the set of all 3×3 symmetric matrices, all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

32. The number of matrices in $\mathscr{A}$ is

(A) 12.
(B) 6.
(C) 9.
(D) 3.

33. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

has a unique solution, is

(A) less than 4.
(B) at least 4 but less than 7.
(C) at least 7 but less than 10.
(D) at least 10.

34. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

is inconsistent, is

(A) 0.
(B) more than 2.
(C) $9x^2 2$.
(D) 1.

[JEE 2009]

Paragraph for Question Nos. 35 to 37

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

35. The probability that $X = 3$ equals

(A) $\frac{25}{216}$. (B) $\frac{25}{36}$. (C) $\frac{5}{36}$. (D) $\frac{125}{216}$.

36. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$. (B) $\frac{25}{36}$. (C) $\frac{5}{36}$. (D) $\frac{25}{216}$.

37. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$. (B) $\frac{25}{216}$. (C) $\frac{5}{36}$. (D) $\frac{25}{36}$.

[JEE 2009]

Paragraph for Question Nos. 38 to 40

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real-valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

38. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$. (B) $-\frac{4\sqrt{2}}{7^3 3^2}$. (C) $\frac{4\sqrt{2}}{7^3 3}$. (D) $-\frac{4\sqrt{2}}{7^3 3}$.

39. The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\displaystyle\int_a^b \frac{x}{3[(f(x))^2 - 1]}\,dx + bf(b) - af(a)$.

(B) $\displaystyle-\int_a^b \frac{x}{3[(f(x))^2 - 1]}\,dx + bf(b) - af(a)$.

(C) $\displaystyle\int_a^b \frac{x}{3[(f(x))^2 - 1]}\,dx - bf(b) + af(a)$.

(D) $\displaystyle-\int_a^b \frac{x}{3[(f(x))^2 - 1]}\,dx - bf(b) + af(a)$.

40. $\displaystyle\int_{-1}^{1} g'(x)\,dx =$

(A) $2\,g(-1)$. (B) 0. (C) $-2\,g(1)$. (D) $2\,g(1)$.

[JEE 2008]

Paragraph for Question Nos. 41 to 43

Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \quad 0 < a < 2.$$

41. Which of the following is true?

(A) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$.

(B) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$.

(C) $f'(1)f'(-1) = (2-a)^2$.

(D) $f'(1)f'(-1) = -(2+a)^2$.

42. Which of the following is true?

(A) $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$.

(B) $f(x)$ is increasing on $(-1, 1)$ and has a local maximum at $x = 1$.

(C) $f(x)$ is increasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$.

(D) $f(x)$ is decreasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$.

43. Let

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}\, dt.$$

Which of the following is true?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$.

(B) $g'(x)$ is negative on $(-\infty, 0)$ and negative on $(0, \infty)$.

(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$.

(D) $g'(x)$ does not change sign on $(-\infty, \infty)$.

[JEE 2008]

Paragraph for Question Nos. 44 to 46

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2},$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}.$$

44. The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\hat{\imath} + 7\hat{\jmath} + 7\hat{k}}{\sqrt{99}}$. (B) $\frac{-\hat{\imath} - 7\hat{\jmath} + 5\hat{k}}{5\sqrt{3}}$. (C) $\frac{-\hat{\imath} + 7\hat{\jmath} + 5\hat{k}}{5\sqrt{3}}$. (D) $\frac{7\hat{\imath} - 7\hat{\jmath} - \hat{k}}{\sqrt{99}}$.

45. The shortest distance between L_1 and L_2 is

(A) 0. (B) $\frac{17}{\sqrt{3}}$. (C) $\frac{41}{5\sqrt{3}}$. (D) $\frac{23}{\sqrt{75}}$.

46. The distance of the point $(1, 1, 1)$ from the plane passing through the point $(-1, -2, -1)$ and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$. (B) $\frac{7}{\sqrt{75}}$. (C) $\frac{13}{\sqrt{75}}$. (D) $\frac{23}{\sqrt{75}}$.

[JEE 2008]

Paragraph for Question Nos. 47 to 49

If a continuous function f defined on the real line $\mathbb{R}$, assumes positive and negative values in $\mathbb{R}$, then the equation $f(x) = 0$ has a root in $\mathbb{R}$. For example, if it is known that a continuous function f on $\mathbb{R}$ is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in $\mathbb{R}$.
Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

47. The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

(A) no point. (B) one point.

(C) two points. (D) more than two points.

48. The positive value of k for which $ke^x - x = 0$ has only one root is

(A) $\frac{1}{e}$. (B) 1.

(C) e. (D) $\log_e 2$.

49. For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

(A) $\left(0, \frac{1}{e}\right)$. (B) $\left(\frac{1}{e}, 1\right)$.

(C) $\left(\frac{1}{e}, \infty\right)$. (D) $(0, 1)$.

[JEE 2007]

Answers

1. (A)	**2.** (C)	**3.** (B)	**4.** (B)
5. (C)	**6.** (A)	**7.** (B)	**8.** (B)
9. (C)	**10.** (A), (B)	**11.** (C), (D)	**12.** (A), (B), (C)
13. (C), (D)	**14.** (A)	**15.** (D)	**16.** (B)
17. (C)	**18.** (A)	**19.** (D)	**20.** (D)
21. (C)	**22.** (B)	**23.** (C)	**24.** (D)
25. (A)	**26.** (B)	**27.** (B)	**28.** (D)
29. (D)	**30.** (C)	**31.** (D)	**32.** (A)
33. (B)	**34.** (B)	**35.** (A)	**36.** (B)
37. (D)	**38.** (B)	**39.** (A)	**40.** (D)
41. (A)	**42.** (A)	**43.** (B)	**44.** (B)
45. (D)	**46.** (C)	**47.** (B)	**48.** (A)
49. (A)			

Matching-List & Matching-Matrix Type

Matching-List Type

Each question in this section has two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D), out of which one is correct.

1. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \dfrac{x}{x-1} > 0\right\}$, $E_2 = \left\{x \in E_1 : \sin^{-1}\left[\log_e\left(\dfrac{x}{x-1}\right)\right] \text{ is a real number}\right\}$.

$\Big($Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.$\Big)$

Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\dfrac{x}{x-1}\right)$ and $g : E_2 \to \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left[\log_e\left(\dfrac{x}{x-1}\right)\right]$.

	List-I		**List-II**
(P)	The range of f is	(1)	$\left(-\infty, \dfrac{1}{1-e}\right] \cup \left[\dfrac{e}{e-1}, \infty\right)$
(Q)	The range of g contains	(2)	$(0, 1)$
(R)	The domain of f contains	(3)	$\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
(S)	The domain of g is	(4)	$(-\infty, 0) \cup (0, \infty)$
		(5)	$\left(-\infty, \dfrac{e}{e-1}\right]$
		(6)	$(-\infty, 0) \cup \left(-\dfrac{1}{2}, \dfrac{e}{e-1}\right]$

The correct option is:

(A) $P \to 4$; $Q \to 2$; $R \to 1$; $S \to 1$.
(B) $P \to 3$; $Q \to 3$; $R \to 6$; $S \to 5$.
(C) $P \to 4$; $Q \to 2$; $R \to 1$; $S \to 6$.
(D) $P \to 4$; $Q \to 3$; $R \to 6$; $S \to 5$.

[JEE 2018]

2. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f_3 : \left(-1, e^{\pi/2} - 2\right) \to \mathbb{R}$ and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$,

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

List-I	List-II
(P) The function f_1 is	(1) **NOT** continuous at $x = 0$
(Q) The function f_2 is	(2) continuous at $x = 0$ and **NOT** differentiable at $x = 0$
(R) The function f_3 is	(3) differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
(S) The function f_4 is	(4) differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is:

(A) $P \to 2;\ Q \to 3;\ R \to 1;\ S \to 4.$
(B) $P \to 4;\ Q \to 1;\ R \to 2;\ S \to 3.$
(C) $P \to 4;\ Q \to 2;\ R \to 1;\ S \to 3.$
(D) $P \to 2;\ Q \to 1;\ R \to 4;\ S \to 3.$

[JEE 2018]

3. Match the following two lists:

List-I		List-II	
(P)	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	(1)	8
(Q)	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	(2)	2
(R)	$\int_{-2}^{2} \frac{3x^2}{1+e^x}\,dx$ equals	(3)	4
(S)	$\dfrac{\int_{-1/2}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx}{\int_{0}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx}$ equals	(4)	0

	(P)	(Q)	(R)	(S)
(A)	(3)	(2)	(4)	(1)
(B)	(2)	(3)	(4)	(1)
(C)	(3)	(2)	(1)	(4)
(D)	(2)	(3)	(1)	(4)

[JEE 2014]

4. Match the following two lists:

List-I	**List-II**
(P) Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\dfrac{\sqrt{3}}{2}$. Then $\dfrac{1}{y(x)}\left\{(x^2-1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}\right\}$ equals	(1) 1
(Q) Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left\lvert\sum_{k=1}^{n-1}(\overrightarrow{a_k} \times \overrightarrow{a_{k+1}})\right\rvert = \left\lvert\sum_{k=1}^{n-1}(\overrightarrow{a_k} \cdot \overrightarrow{a_{k+1}})\right\rvert$, then the minimum value of n is	(2) 2
(R) If the normal from the point $P(h, 1)$ on the ellipse $\dfrac{x^2}{6} + \dfrac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	(3) 8
(S) Number of positive solutions satisfying the equation $\tan^{-1}\left(\dfrac{1}{2x+1}\right) + \tan^{-1}\left(\dfrac{1}{4x+1}\right) = \tan^{-1}\left(\dfrac{2}{x^2}\right)$ is	(4) 9

	(P)	(Q)	(R)	(S)
(A)	(4)	(3)	(2)	(1)
(B)	(2)	(4)	(3)	(1)
(C)	(4)	(3)	(1)	(2)
(D)	(2)	(4)	(1)	(3)

[JEE 2014]

5. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : [0, \infty) \to \mathbb{R}$, $f_3 : \mathbb{R} \to \mathbb{R}$ and $f_4 : \mathbb{R} \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$$

$$f_2(x) = x^2;$$

$$f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0 \end{cases}$$

and
$$f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0. \end{cases}$$

List-I	List-II
(P) f_4 is	(1) onto but not one-one
(Q) f_3 is	(2) neither continuous nor one-one
(R) $f_2 \circ f_1$ is	(3) differentiable but not one-one
(S) f_2 is	(4) continuous and one-one

	(P)	(Q)	(R)	(S)
(A)	(3)	(1)	(4)	(2)
(B)	(1)	(3)	(4)	(2)
(C)	(3)	(1)	(2)	(4)
(D)	(1)	(3)	(2)	(4)

[JEE 2014]

6. A line $L : y = mx + 3$ meets y-axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

List-I	**List-II**
(P) $m =$	(1) $\frac{1}{4}$
(Q) Maximum area of ΔEFG is	(2) 4
(R) $y_0 =$	(3) 2
(S) $y_1 =$	(4) 1

	(P)	(Q)	(R)	(S)
(A)	(4)	(1)	(2)	(3)
(B)	(3)	(4)	(1)	(2)
(C)	(1)	(3)	(2)	(4)
(D)	(1)	(3)	(4)	(2)

[JEE 2013]

7. Match the following two lists:

	List-I		List-II
(P)	$\left[\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y) + y\ \sin(\tan^{-1}y)}{\cot(\sin^{-1}y) + \tan(\sin^{-1}y)}\right)^2 + y^4\right]^{1/2}$ takes value	(1)	$\frac{1}{2}\sqrt{\frac{5}{3}}$
(Q)	If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$, then possible value of $\cos\left(\frac{x-y}{2}\right)$ is	(2)	$\sqrt{2}$
(R)	If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x\ \sin 2x\ \sec x$ $= \cos x\ \sin 2x\ \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$, then possible value of $\sec x$ is	(3)	$\frac{1}{2}$
(S)	If $\cot \sin^{-1}\sqrt{1 - x^2} = \sin \tan^{-1}(x\sqrt{6})$, $x \neq 0$, then the possible value of x is	(4)	1

	(P)	(Q)	(R)	(S)
(A)	(4)	(3)	(1)	(2)
(B)	(4)	(3)	(2)	(1)
(C)	(3)	(4)	(2)	(1)
(D)	(3)	(4)	(1)	(2)

[JEE 2013]

8. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

List-I	**List-II**
(P) $a =$	(1) 13
(Q) $b =$	(2) -3
(R) $c =$	(3) 1
(S) $d =$	(4) -2

	(P)	(Q)	(R)	(S)
(A)	(3)	(2)	(4)	(1)
(B)	(1)	(3)	(4)	(2)
(C)	(3)	(2)	(1)	(4)
(D)	(2)	(4)	(1)	(3)

[JEE 2013]

9. Match the following two lists:

List-I	List-II
(P) Volume of parallelepiped determined by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is 2. Then, the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b})$, $3(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ is	(1) 100
(Q) Volume of parallelepiped determined by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is 5. Then, the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b})$, $(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(2) 30
(R) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then, the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is	(3) 24
(S) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then, the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(4) 60

	(P)	(Q)	(R)	(S)
(A)	(4)	(2)	(3)	(1)
(B)	(2)	(3)	(1)	(4)
(C)	(3)	(4)	(1)	(2)
(D)	(1)	(4)	(3)	(2)

[JEE 2013]

Matching-Matrix Type

In this section, each question contains statements given in two columns, which have to be matched. The statements in Column I are labelled (A), (B), (C) and (D), while the statements in Column II are labelled (p), (q), (r), (s) and (t). Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened.

10. Match the following two lists:

List-I	List-II
(A) In $\mathbb{R}^2$, if the magnitude of the projection vector of the vector $\alpha\hat{i} + \beta\hat{j}$ on $\sqrt{3}\,\hat{i} + \hat{j}$ is $\sqrt{3}$ and if $\alpha = 2 + \sqrt{3}\,\beta$, then possible value(s) of $\lvert\alpha\rvert$ is(are)	(p) 1
(B) Let a and b be real numbers such that the function $f(x) = \begin{cases} -3ax^2 - 2 & \text{if } x < 1 \\ bx + a^2 & \text{if } x \geq 1 \end{cases}$ is differentiable for all $x \in \mathbb{R}$. Then possible value(s) of a is(are)	(q) 2
(C) Let $\omega \neq 1$ be a complex cube root of unity. If $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$, then possible value(s) of n is(are)	(r) 3
(D) Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $\lvert q - a\rvert$ is(are)	(s) 4
	(t) 5

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2015]

11. Match the following two lists:

	List-I		**List-II**
(A)	In a triangle XYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \dfrac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda) = 0$ is(are)	(p)	1
(B)	In a triangle XYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\dfrac{a}{b}$ is(are)	(q)	2
(C)	In $\mathbb{R}^2$, let $\sqrt{3}\,\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\,\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\dfrac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is(are)	(r)	3
(D)	Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = \lvert\alpha x - 1\rvert + \lvert\alpha x - 2\rvert + \alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha) + \dfrac{8\sqrt{2}}{3}$, when $\alpha = 0$ and $\alpha = 1$, is(are)	(s)	5
		(t)	6

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2015]

12. Match the following two lists:

List-I	List-II
(A) The set $\left\{ \operatorname{Re}\left(\dfrac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1, z \neq \pm 1 \right\}$ is	(p) $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\dfrac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q) $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \le \theta < \dfrac{\pi}{2} \right\}$ is	(r) $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x - 10)$, $x \geq 0$, then $f(x)$ is increasing in	(s) $(-\infty, -1] \cup [1, \infty)$
	(t) $(-\infty, 0] \cup [2, \infty)$

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2011]

13. Match the following two lists:

List-I	**List-II**
(A) A line from the origin meets the lines $\dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z+1}{1}$ and $\dfrac{x-\frac{8}{3}}{2}=\dfrac{y+3}{-1}=\dfrac{z-1}{1}$ at P and Q respectively. If length $PQ=d$, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\dfrac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}=0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function on $[-\pi,\pi]$ given by $f(0)=9$ and $f(x)=\dfrac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$ for $x\neq 0$. The value of $\dfrac{2}{\pi}\displaystyle\int_{-\pi}^{\pi}f(x)\,dx$ is	(s) 5
	(t) 6

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2010]

14. Match the following two lists:

List-I		**List-II**	
(A)	Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$	(p)	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
(B)	Interval containing the value of the integral $\displaystyle\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q)	$\left(0, \dfrac{\pi}{2}\right)$
(C)	Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r)	$\left(\dfrac{\pi}{8}, \dfrac{5\pi}{4}\right)$
(D)	Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s)	$\left(0, \dfrac{\pi}{8}\right)$
		(t)	$(-\pi, \pi)$

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2009]

15. Match the following two lists:

List-I		List-II	
(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left\lfloor \frac{6x}{\pi} \right\rfloor \cos\left(\left\lfloor \frac{3x}{\pi} \right\rfloor\right)$, where $\lfloor y \rfloor$ denotes the largest integer less than or equal to y	(q)	$\frac{\pi}{4}$
(C)	Volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r)	$\frac{\pi}{3}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(s)	$\frac{\pi}{2}$
		(t)	π

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2009]

16. Match the following two lists:

List-I	**List-II**
(A) The number of solutions of the equation $x\,e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of $y(\ln 2)$	(s) 4
	(t) 5

	(p)	(q)	(r)	(s)	(t)
(A)	p	q	r	s	t
(B)	p	q	r	s	t
(C)	p	q	r	s	t
(D)	p	q	r	s	t

[JEE 2009]

Matching-Matrix Type

In this section, each question contains statements given in two columns, which have to be matched. The statements in Column I are labelled (A), (B), (C) and (D), while the statements in Column II are labelled (p), (q), (r) and (s). Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened.

17. Consider the lines given by

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

List-I	**List-II**
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(r) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2008]

18. Match the following two lists:

List-I	**List-II**
(A) The minimum value of $\dfrac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t=(-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a=\log_3(\log_3 2)$. An integer k satisfying $1<2^{(-k+3^{-\alpha})}<2$, must be less than	(r) 2
(D) If $\sin\theta=\cos\varphi$, then the possible values of $\dfrac{1}{\pi}\left(\theta\pm\varphi-\dfrac{\pi}{2}\right)$	(s) 3

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2008]

19. In the following $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

List-I		**List-II**	
(A)	$x\|x\|$	(p)	continuous in $(-1, 1)$
(B)	$\sqrt{\|x\|}$	(q)	differentiable in $(-1, 1)$
(C)	$x + \lfloor x \rfloor$	(r)	strictly increasing in $(-1, 1)$
(D)	$\|x-1\| + \|x+1\|$	(s)	not differentiable at least at one point in $(-1, 1)$

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2007]

20. Consider the following:

List-I		**List-II**	
(A)	$\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$	(p)	$\dfrac{1}{2}\log\left(\dfrac{2}{3}\right)$
(B)	$\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$	(q)	$\dfrac{\pi}{4}$
(C)	$\displaystyle\int_2^3 \frac{1}{1-x^2}\,dx$	(r)	$\dfrac{\pi}{3}$
(D)	$\displaystyle\int_1^2 \frac{1}{x\sqrt{x^2-1}}\,dx$	(s)	$\dfrac{\pi}{2}$

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2007]

21. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$.

List-I	**List-II**
(A) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2007]

22. Let (x, y) be such that

$$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}.$$

List-I	**List-II**
(A) If $a = 1$ and $b = 0$, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1$ and $b = 1$, then (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	(r) lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

	(p)	(q)	(r)	(s)
(A)	p	q	r	s
(B)	p	q	r	s
(C)	p	q	r	s
(D)	p	q	r	s

[JEE 2007]

Answers

1. (A) **2.** (D) **3.** (D) **4.** (A)

5. (D) **6.** (D) **7.** (B) **8.** (A)

9. (C)

10.

	(p)	(q)	(r)	(s)	(t)
(A)	●	●	○	○	○
(B)	●	●	○	○	○
(C)	●	●	○	●	●
(D)	○	●	○	○	●

11.

	(p)	(q)	(r)	(s)	(t)
(A)	●	○	●	●	○
(B)	●	○	○	○	○
(C)	●	●	○	○	○
(D)	○	○	○	●	●

12.

	(p)	(q)	(r)	(s)	(t)
(A)	○	○	○	●	○
(B)	○	○	○	○	●
(C)	○	○	●	○	○
(D)	○	○	●	○	○

13.

	(p)	(q)	(r)	(s)	(t)
(A)	○	○	○	○	●
(B)	●	○	●	○	○
(C)	○	●	○	●	○
(D)	○	○	●	○	○

14.

	(p)	(q)	(r)	(s)	(t)
(A)	●	●	○	●	○
(B)	●	○	○	○	●
(C)	●	●	●	○	●
(D)	○	○	○	●	○

15.

	(p)	(q)	(r)	(s)	(t)
(A)	●	●	○	●	○
(B)	●	○	○	○	●
(C)	●	●	●	○	●
(D)	○	○	○	●	○

16.

	(p)	(q)	(r)	(s)	(t)
(A)	●	○	○	○	○
(B)	○	●	○	●	○
(C)	○	●	●	●	●
(D)	○	○	●	○	○

17.

	(p)	(q)	(r)	(s)
(A)	○	○	○	●
(B)	●	●	○	○
(C)	○	○	●	○
(D)	●	●	○	●

18.

	(p)	(q)	(r)	(s)
(A)	p	q	**r**	s
(B)	p	**q**	r	**s**
(C)	p	q	**r**	**s**
(D)	**p**	q	**r**	s

19.

	(p)	(q)	(r)	(s)
(A)	**p**	**q**	**r**	s
(B)	**p**	q	r	**s**
(C)	p	q	**r**	**s**
(D)	**p**	**q**	r	s

20.

	(p)	(q)	(r)	(s)
(A)	p	q	r	**s**
(B)	p	q	r	**s**
(C)	**p**	q	r	s
(D)	p	q	**r**	s

21.

	(p)	(q)	(r)	(s)
(A)	**p**	q	**r**	**s**
(B)	p	**q**	r	**s**
(C)	p	**q**	r	**s**
(D)	**p**	q	**r**	**s**

22.

	(p)	(q)	(r)	(s)
(A)	**p**	q	r	s
(B)	p	**q**	r	s
(C)	**p**	q	r	s
(D)	p	q	r	**s**